

# Composing Constraint Automata, State-by-State

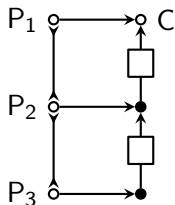
Sung-Shik T.Q. Jongmans<sup>1</sup>   Tobias Kappé<sup>2</sup>   Farhad Arbab<sup>1,2</sup>

<sup>1</sup>Centrum Wiskunde & Informatica, Amsterdam

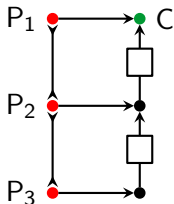
<sup>2</sup>Leiden Institute of Advanced Computer Science, Leiden

Formal Aspects of Component Software, 2015

Reo is a *coordination language*, it facilitates coordinating synchronization and communication among components.

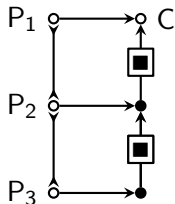


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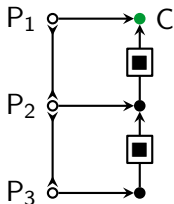
**input** / **output**

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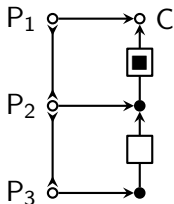
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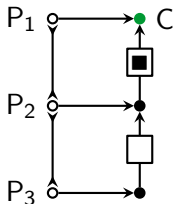
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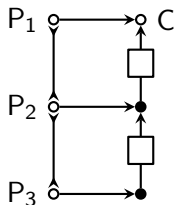
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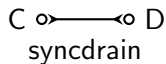
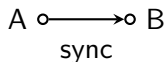
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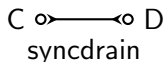
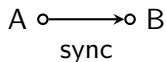
# Constraint Automata

Constraint Automata provide a very useful semantics for Reo.



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$\{C, D\}, \top$



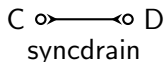
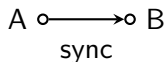
$\{E\}, d(E) = x'$



$\{F\}, d(F) = x$

# Constraint Automata

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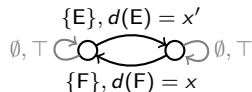


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# Constraint Automata

Let  $q_1 \xrightarrow{P_1, \phi_1} q'_1$  and  $q_2 \xrightarrow{P_2, \phi_2} q'_2$  be transitions of  $\alpha_1$  and  $\alpha_2$  respectively. If

$$P_1 \cap \text{Port}(\alpha_2) = P_2 \cap \text{Port}(\alpha_1)$$

then  $(q_1, q_2) \xrightarrow{P_1 \cup P_2, \phi_1 \wedge \phi_2} (q'_1, q'_2)$  is a transition of  $\alpha_1 \otimes \alpha_2$ .

Informally: *transitions can be composed if (and only if) they agree on common ports.*

# Constraint Automata

As an example, let's compute the following composition:

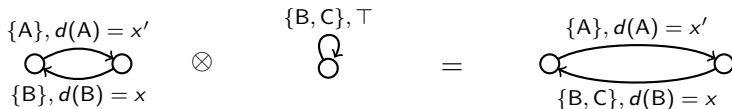
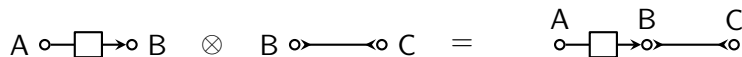
$$A \circ \square \rightarrow B \quad \otimes \quad B \circ \longleftarrow C \quad =$$

---

$$\begin{array}{l} \{A\}, d(A) = x' \\ \circ \rightleftarrows \circ \\ \{B\}, d(B) = x \end{array} \quad \otimes \quad \begin{array}{l} \{B, C\}, T \\ \circ \curvearrowright \circ \end{array} \quad =$$

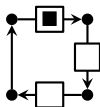
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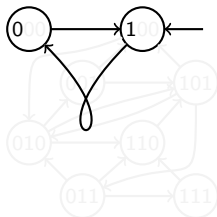
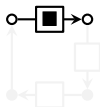
# Problematic products

Computation of total semantics sometimes gives rise to exponential growth of intermediary products, such as seen here:



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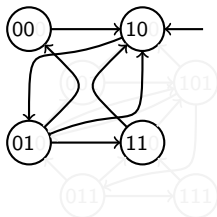
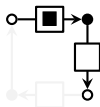
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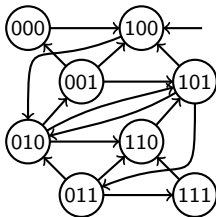
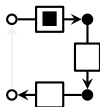
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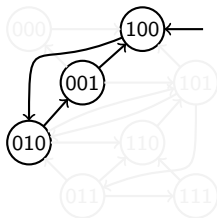
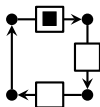
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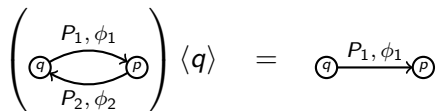
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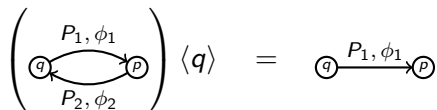
$$\left( \begin{array}{c} \text{---} P_1, \phi_1 \text{---} \\ \text{---} P_2, \phi_2 \text{---} \\ q \rightleftarrows p \end{array} \right) \langle q \rangle = q \xrightarrow{P_1, \phi_1} p$$

We write  $\sqcup A$  for the *state-based recomposition* of Constraint Automata  $\alpha \in A$ .

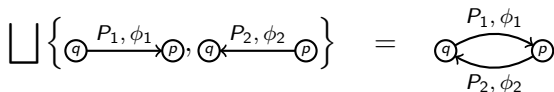
$$\sqcup \left\{ \begin{array}{c} \text{---} P_1, \phi_1 \text{---} \\ q \rightarrow p \end{array} , \begin{array}{c} \text{---} P_2, \phi_2 \text{---} \\ q \leftarrow p \end{array} \right\} = \begin{array}{c} \text{---} P_1, \phi_1 \text{---} \\ q \rightleftarrows p \\ \text{---} P_2, \phi_2 \text{---} \end{array}$$

# State-by-State Product

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A decomposed automaton can be recomposed to obtain the original:

$$\sqcup \{ \alpha\langle q \rangle : q \in \text{State}(\alpha) \} = \alpha$$

# State-by-State Product

More importantly, we show that the product distributes over decomposition:

$$(\alpha_1 \otimes \cdots \otimes \alpha_n) \langle (q_1, \dots, q_n) \rangle = \alpha_1 \langle q_1 \rangle \otimes \cdots \otimes \alpha_n \langle q_n \rangle$$

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As in:

$$\left( \begin{array}{c} \text{Automaton 1} \\ \text{Automaton 2} \end{array} \right) \otimes \langle (q, r) \rangle = \left( \begin{array}{c} \text{Automaton 1} \\ \text{State } q \end{array} \right) \otimes \left( \begin{array}{c} \text{Automaton 2} \\ \text{State } r \end{array} \right)$$



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This means that we can cheaply calculate part of the composition!

Idea: use this to calculate *only the reachable part* of the composition.

# State-by-State Product

Algorithm “computes transitions that exit reachable states”:

$A := \emptyset$

$A' :=$

$\{\alpha_1 \langle q_1 \rangle \otimes \cdots \otimes \alpha_n \langle q_n \rangle : (q_1, \dots, q_n) \in \text{Init}(\alpha_1) \times \cdots \times \text{Init}(\alpha_n)\}$

**while**  $\alpha \in A' \setminus A$  **for some**  $\alpha$  **do**

$A := A \cup \{\alpha\}$

$A' :=$

$A' \cup \{\alpha_1 \langle q'_1 \rangle \otimes \cdots \otimes \alpha_n \langle q'_n \rangle : (q, P, \phi, (q'_1, \dots, q'_n)) \in \text{Tr}(\alpha)\}$

**end while**

We show that, after the loop,  $\bigsqcup A = \lfloor \alpha_1 \otimes \cdots \otimes \alpha_n \rfloor$ .

# Implementation

We implemented the algorithm in Extensible Coordination Tools, a toolchain for Reo.

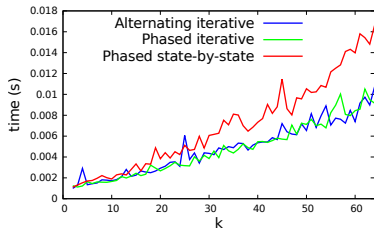
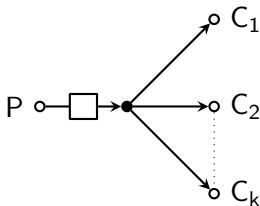
We then compared the composition speed of some circuits parameterized in  $k$ .

Three methods were compared:

- ▶ *Alternating iterative*: composition and abstraction alternated
- ▶ *Phased iterative*: abstraction occurs after composition
- ▶ *Phased state-by-state*: by our algorithm, abstraction occurs after composition

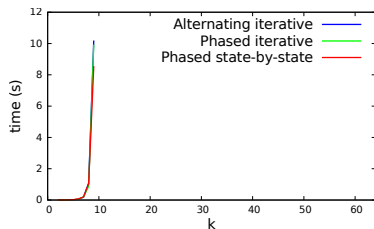
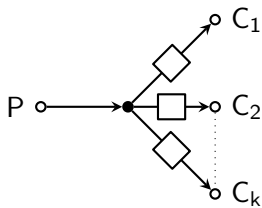
# Implementation

For some circuits, there was no big difference, because the state space is small . . .



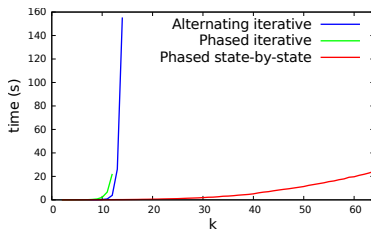
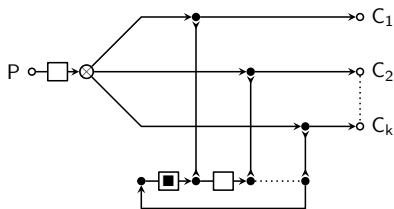
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... or because the state space is large.



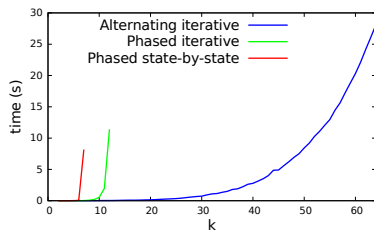
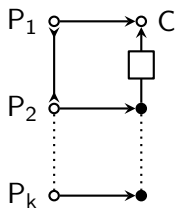
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For others, the new algorithm was faster because of a small *reachable* state space.



# Implementation

And in one case, abstraction turned out to be important to limit the state space:



# Conclusion

- ▶ New method makes composition of some circuits feasible.
- ▶ Method is applicable to other formalisms that use Constraint Automata.
- ▶ Some problematic cases remain, notably with relation to abstraction.
- ▶ Future research could look into a heuristic as to which method to use.