CF-GKAT

Efficient Validation of Control-Flow Transformations

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Program equivalence

```
if x = 1 then
  | y++;
else
  | z := z/2;
end
```

$\equiv$

```
if x ≠ 1 then
  | z := z/2;
else
  | y++;
end
```

Propositional program equivalence

```
if b then
|   p;
else
|   q;
end
```

$\equiv$

```
if not b then
|   q;
else
|   p;
end
```

Propositional program equivalence

```
s;
while b do
  p;
end
while c do
  q;
  while b do
    p;
  end
end
```

$\equiv$

```
s;
while b or c do
  if b then
    p;
  else
    q;
  end
end
```

Kleene Algebra with Tests (KAT)

Fix a set $\{p, q, \ldots\}$ of actions and a Boolean algebra $\{b, c, \ldots\}$ of tests

$$b \mid p \mid e + f \mid e; f \mid e^*$$

- We can embed simple propositional programs:

$$\boxed{\text{if } b \text{ then } e \text{ else } f} = b; e + (\text{not } b); f$$

$$\boxed{\text{while } b \text{ do } e} = (b; e)^* \,; (\text{not } b)$$

- Language semantics in terms of *guarded strings*.
- Complete and finitary axiomatization
- Non-determinism makes equivalence PSPACE-complete
  - Basically: translate to automaton and check bisimilarity

(Kozen 1996), (Kozen & Smith 1996)

Guarded Kleene Algebra with Tests (GKAT)

Fix a set $\{p, q, \ldots\}$ of actions and a Boolean algebra $\{b, c, \ldots\}$ of tests

$$b \mid p \mid e +_b f \mid e; f \mid e^{(b)}$$

- Language semantics in terms of *guarded strings*.
- Complete but infinitary axiomatization (open problem)
- Language equivalence is efficiently decidable! (nearly linear)
  - Same idea as in KAT, but different composition operators.

(Kozen & Tseng 2008), (Smolka et al. 2020)

GKAT semantics

Guarded strings

Inherited from KAT: sets of *guarded strings*, i.e., words of the form

$$\alpha_1 p_1 \alpha_2 p_2 \alpha_3 p_3 \cdots \alpha_n p_n \alpha_{n+1}$$

where the *atom* $\alpha_i \in At$ tells us the truth value of each test at point i.

GKAT semantics

By example

$$\left[\!\left[\begin{array}{l}\textbf{if } b \textbf{ then} \\ \quad p; \\ \textbf{else} \\ \quad q; \\ \textbf{end}\end{array}\right]\!\right] = \left\{\begin{array}{l} bpb, \\ bp\overline{b}, \\ \overline{b}q\overline{b}, \\ \overline{b}qb \end{array}\right\}$$

GKAT semantics

By example

$$
\left\llbracket
\begin{array}{l}
\textbf{while } b \textbf{ or } c \textbf{ do} \\
\quad \textbf{if } b \textbf{ then} \\
\quad\quad p; \\
\quad \textbf{else} \\
\quad\quad q; \\
\quad \textbf{end} \\
\textbf{end}
\end{array}
\right\rrbracket
=
\left\{
\begin{array}{l}
\bar{b}\bar{c}, \\
\bar{b}cq\bar{b}\bar{c}, \\
b\bar{c}p\bar{b}cq\bar{b}\bar{c}, \\
\dots
\end{array}
\right\}
$$

GKAT decision procedure

GKAT automata

This comes from (Smolka et al. 2020).

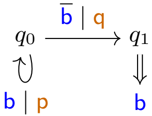
Let Q be a set. The set of *GKAT dynamics* on Q, denoted Q, consists of functions

$$At \to \bot + \checkmark + \Sigma \times Q$$

A *GKAT automaton* is a pointed G-coalgebra.

That is, it is a tuple (Q, δ, q_0) where:

- Q is a (finite) set of states;
- $q_0 \in Q$ is the *initial state*; and
- $\delta : Q \to GQ$ is the *transition function*.



Straightforward semantics in terms of guarded languages.

GKAT decision procedure

Translating to GKAT automata

```
while b or c do
    if b then
        p;
    else
        q;
    end
end
```

$\mapsto$

$q_0 \Longrightarrow \overline{b}\,\overline{c}$

$\circlearrowleft$

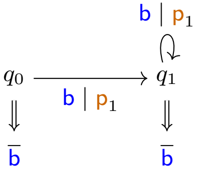
$bc \mid p$

$b\overline{c} \mid p$

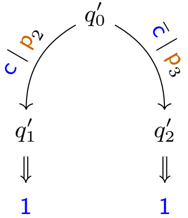
$\overline{b}c \mid q$

GKAT decision procedure

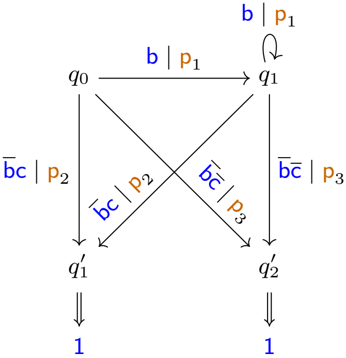
Translating to GKAT automata



$e = $ **while** b **do** p_1



$f = $ **if** c **then** p_2 **else** p_3



$g = e \cdot f$

Theorem

For every GKAT expression e, we can construct a GKAT automaton A_e such that $[\![e]\!] = [\![A_e]\!]$.

This is a basic syntax-directed translation, à la (Thompson 1968).

Lemma

A_e is at most linear in the size of e.

Equivalence (bisimilarity) of GKAT automata can be easily decided.

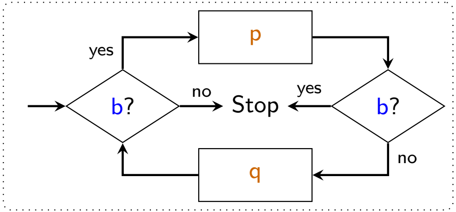
Goto removal

```
L:
if b then
    p;
    goto R;
return;
R:
if not b then
    q;
    goto L;
return;
```

goto removal
(e.g., calipso)
→

```
x := 1;
while x ≠ 0 do
    if x = 1 and b then
        p;
        x := 2;
    else if x = 2 and
     not b then
        q;
        x := 1;
    else
        x := 0;
end
```

Decompilation



GHIDRA

```
while b do
    p;
    if b then
        break;
    q;
end
```

Map of the Problematique

- GKAT does not support labels, **goto**, **return**, **break**, variables...
- It cannot even express the example programs! (Schmid, K. & Silva 2021)
- They *can* be translated to KAT expressions (Kozen 2008), e.g.:

$$(\mathsf{b}\mathsf{p}\overline{\mathsf{b}}\mathsf{q})^*(\mathsf{b}\mathsf{p}\mathsf{b} + \overline{\mathsf{b}})$$

- This is a rather non-trivial process...
- ...and KAT equivalence is PSPACE-hard...
- ...and really, we just need an automaton.

Enter CF-GKAT

Fix sets of:

- *actions* $\mathsf{p} \in \Sigma$;
- *tests* $\mathsf{b} \in T$;
- *labels* $\ell \in L$; and
- *indicator values* $i \in I$.

CF-GKAT is GKAT augmented with non-local flow control:

$$Exp \ni \mathsf{e}, \mathsf{f} ::= \textbf{assert } \mathsf{b} \mid \mathsf{p} \in \Sigma \mid x := i \ (i \in I) \mid \mathsf{e}; \mathsf{f} \mid \textbf{if } \mathsf{b} \textbf{ then } \mathsf{e} \textbf{ else } \mathsf{f} \mid$$
$$\textbf{while } \mathsf{b} \textbf{ do } \mathsf{e} \mid \textbf{break} \mid \textbf{return} \mid \textbf{goto } \ell \ (\ell \in L) \mid \textbf{label } \ell \ (\ell \in L)$$

CF-GKAT semantics

Guarded strings with continuations

First step: *guarded strings with continuations*, i.e.,

$$\alpha_1 p_1 \alpha_2 p_2 \alpha_3 p_3 \cdots \alpha_n p_n \alpha_{n+1} \cdot c$$

where c is a *continuation* of the form:

$$\mathbf{acc}\ i \qquad \mathbf{brk}\ i \qquad \mathbf{ret} \qquad \mathbf{jmp}\ (\ell, i)$$

A continuation tells us how a partial computation can proceed.

CF-GKAT semantics

By example

$$\left[\!\!\left[\begin{array}{l}\mathsf{p};\\ \textbf{if } \mathsf{b} \textbf{ then}\\ \quad \textbf{break};\\ \mathsf{q};\end{array}\right]\!\!\right]^{\sharp}_{i} = \left\{\begin{array}{l}\mathsf{b}\mathsf{p}\overline{\mathsf{b}}\mathsf{q}\overline{\mathsf{b}} \cdot \mathbf{acc}\ i,\\ \mathsf{b}\mathsf{p}\overline{\mathsf{b}}\mathsf{q}\mathsf{b} \cdot \mathbf{acc}\ i,\\ \mathsf{b}\mathsf{p}\mathsf{b} \cdot \mathbf{brk}\ i\end{array}\right\}$$

$$\left[\!\!\left[\begin{array}{l}\textbf{while } \mathsf{b} \textbf{ do}\\ \quad \mathsf{p};\\ \quad \textbf{if } \mathsf{b} \textbf{ then}\\ \quad\quad \textbf{break};\\ \quad \mathsf{q};\\ \textbf{end}\end{array}\right]\!\!\right]^{\sharp}_{i} = \left\{\begin{array}{l}\mathsf{b}\mathsf{p}\overline{\mathsf{b}}\mathsf{q}\overline{\mathsf{b}} \cdot \mathbf{acc}\ i,\\ \mathsf{b}\mathsf{p}\overline{\mathsf{b}}\mathsf{q}\mathsf{b}\mathsf{p}\overline{\mathsf{b}}\mathsf{q}\overline{\mathsf{b}} \cdot \mathbf{acc}\ i,\\ \mathsf{b}\mathsf{p}\mathsf{b} \cdot \mathbf{acc}\ i\\ \mathsf{b}\mathsf{p}\overline{\mathsf{b}}\mathsf{q}\mathsf{b}\mathsf{p}\mathsf{b} \cdot \mathbf{acc}\ i\\ \dots\end{array}\right\}$$

CF-GKAT semantics

By example

$$\left[\!\left[\begin{array}{l} \texttt{L}: \\ \textbf{if } \mathsf{b} \textbf{ then} \\ \quad \mathsf{p}; \\ \quad \textbf{goto } \texttt{R}; \\ \textbf{return}; \\ \texttt{R}: \\ \textbf{if not } \mathsf{b} \textbf{ then} \\ \quad \mathsf{q}; \\ \quad \textbf{goto } \texttt{L}; \\ \textbf{return}; \end{array}\right]\!\right]^{\sharp}_{i} = \left\{ \begin{array}{l} \mathsf{bpb} \cdot \textbf{jmp}\ (\texttt{R}, i), \\ \mathsf{bp}\overline{\mathsf{b}} \cdot \textbf{jmp}\ (\texttt{R}, i), \\ \overline{\mathsf{b}} \cdot \textbf{ret} \end{array} \right\}$$

$$\left[\!\left[\begin{array}{l} \texttt{L}: \\ \textbf{if } \mathsf{b} \textbf{ then} \\ \quad \mathsf{p}; \\ \quad \textbf{goto } \texttt{R}; \\ \textbf{return}; \\ \texttt{R}: \\ \textbf{if not } \mathsf{b} \textbf{ then} \\ \quad \mathsf{q}; \\ \quad \textbf{goto } \texttt{L}; \\ \textbf{return}; \end{array}\right]\!\right]^{\texttt{R}}_{i} = \left\{ \begin{array}{l} \overline{\mathsf{b}}\mathsf{qb} \cdot \textbf{jmp}\ (\texttt{L}, i), \\ \overline{\mathsf{b}}\mathsf{q}\overline{\mathsf{b}} \cdot \textbf{jmp}\ (\texttt{L}, i), \\ \mathsf{b} \cdot \textbf{ret} \end{array} \right\}$$

CF-GKAT semantics

Resolution

Now we have a semantics of the form

$$[\![-]\!] : L \to I \to \text{guarded languages with continuations}$$

where guarded words end at the first **goto** statement.

What we need is a semantics of the form

$$[\![-]\!]{\downarrow} : L \to I \to \text{guarded languages}$$

where the jumps are "resolved" by stringing them together.

CF-GKAT semantics

Resolution

Resolution, in symbols:

$$\frac{w \cdot c \in [\![P]\!]_i^\ell \qquad c \in \{\mathbf{acc}\ j, \mathbf{ret}\}}{w \in [\![P]\!]{\downarrow}_i^\ell} \qquad \frac{w\alpha \cdot \mathbf{jmp}\ (\ell', j) \in [\![P]\!]_i^\ell \qquad \alpha x \in [\![P]\!]{\downarrow}_j^{\ell'}}{w\alpha x \in [\![P]\!]{\downarrow}_i^\ell}$$

$P =$

```
L:
if b then
  | p;
  | goto R;
return;
R:
if not b then
  | q;
  | goto L;
return;
```

Then we can infer:

$$\frac{\mathsf{b} \cdot \mathbf{ret} \in [\![P]\!]_i^{\mathtt{R}}}{\mathsf{b} \in [\![P]\!]{\downarrow}_i^{\mathtt{R}}}$$

$$\frac{\mathsf{bpb} \cdot \mathbf{jmp}\ (\mathtt{R}, i) \in [\![P]\!]_i^\sharp \qquad \mathsf{b} \in [\![P]\!]{\downarrow}_i^{\mathtt{R}}}{\mathsf{bpb} \in [\![P]\!]{\downarrow}_i^\sharp}$$

Let C be the set of continuations (e.g., $\mathbf{acc}\ i$, $\mathbf{brk}\ i$, etc).

A *CF-GKAT dynamics* on a set X is a function of the form

$$I \to At \to \bot + C + \Sigma \times X \times I$$

We write GX for the set of CF-GKAT dynamics on X.

A CF-GKAT automaton is a tuple $A = (Q, \delta, q, \lambda)$ where

- Q is a set of *states* with $q \in Q$ the *initial state*
- $\delta : Q \to GQ$ is the *transition* function
- $\lambda : L \to GQ$ is the *jump* function

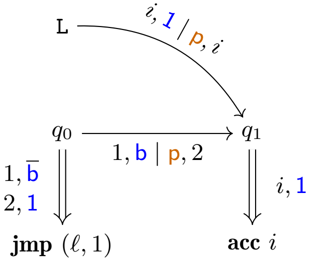
Has a straightforward semantics $[\![A]\!] : L \to I \to$ guarded languages with continuations.

Decision procedure

CF-GKAT automata

if b **and** $x = 1$ **then**
    $x := 2$;
    L:
    p;
else
    $x := 1$;
    goto L;

$\mapsto$



Decision procedure

Translating to CF-GKAT automata

Theorem

For every CF-GKAT expression e, we can construct a CF-GKAT automaton A_e such that $[\![e]\!] = [\![A_e]\!]$.

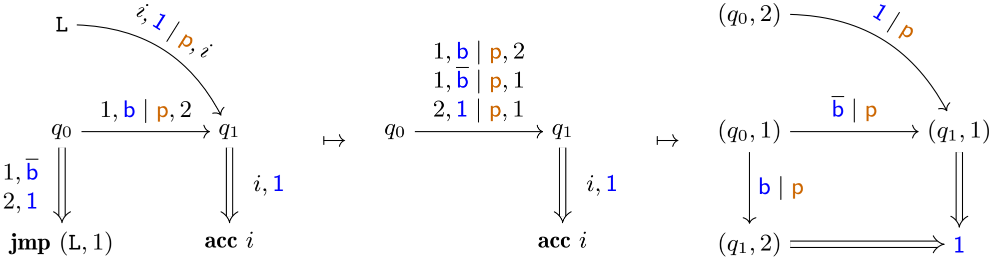
Another syntax-directed translation, à la (Thompson 1968).

Lemma

A_e is at most linear in the size of e.

Decision procedure

Translating to GKAT automata



Decision procedure

Translating to GKAT automata

Theorem

For every CF-GKAT automaton A, we can construct a GKAT automaton $A{\downarrow}$ such that $[\![A]\!]{\downarrow} = [\![A{\downarrow}]\!]$.

Lemma

$A{\downarrow}$ is at most $|I|$ times as large as A

Validation

Blinding

C programs are still not CF-GKAT programs:

- Parsing C can be really hard!
- Same statements mapped to same variable.
- Which variables are indicators?
- Macros may hide relevant information.

LLVM/clang to the rescue

Validation

Blinding

```
#define hidden(x) \
    x=f(x); goto R;

int foo(int x, int y) {
  L:
  if (x == y) {
    x = f(x);
    hidden(x);
  }
  return;
  R:
  if (x != y) {
    y = g(y);
    goto L;
  }
}
```

$\mapsto$

```
L:
if b then
  | p;
  | goto R;
return;
R:
if not b then
  | q;
  | goto L;
```

Validation

Working on `coreutils`

We chose to work on `mp_factor_using_pollard_rho`:

- 91 lines of code (before macro expansion)
- loops nested three levels deep
- has a **break** statement inside
- also contains a **goto** for error handling
- no indicator variables (yet)

Validation

Working on `coreutils`

First experiment:

- Blind, compile (clang), and then decompile (Ghidra).
- Decompiled code has 3 more `goto`'s and labels.
- Some manual effort to remove decompilation artifacts.
- Compare original code to decompiled code: OK.

Validation

Working on `coreutils`

Second experiment:

- Blind, then eliminate **goto** using indicators (Erosa & Hendren 1994, Cassé et al. 2002)
- Some manual effort to make indicator detection work.
- Compare original code to refactored code: OK.

Conclusion & Further Work

- More experiments are necessary!
  - Automated testing pipeline
  - Scale to all of `coreutils`, Linux kernel, ...
  - Extract control flow from binary (no compilation).
- Expand support for top-level syntax:.
  - the **continue** statement
  - multi-way branching using **switch**
  - **do-while** loops with **break**
- Integration with existing decompiler
  - Baked-in translation validation
  - Continuous integration