Formal Abstractions for Packet Scheduling

A. Mohan, Y. Liu, N. Foster, T. Kappé, D. Kozen

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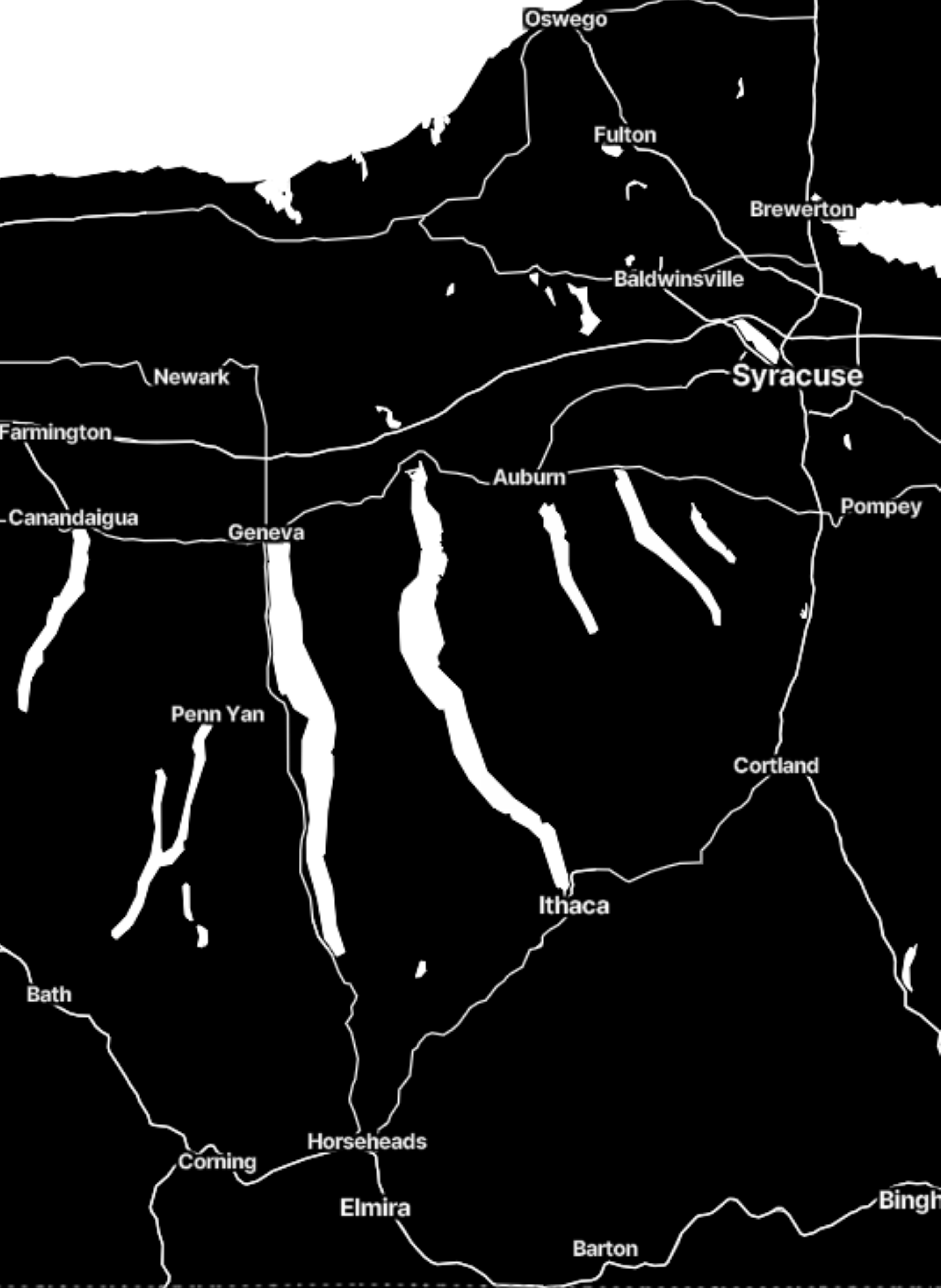
Has let me use his slides!



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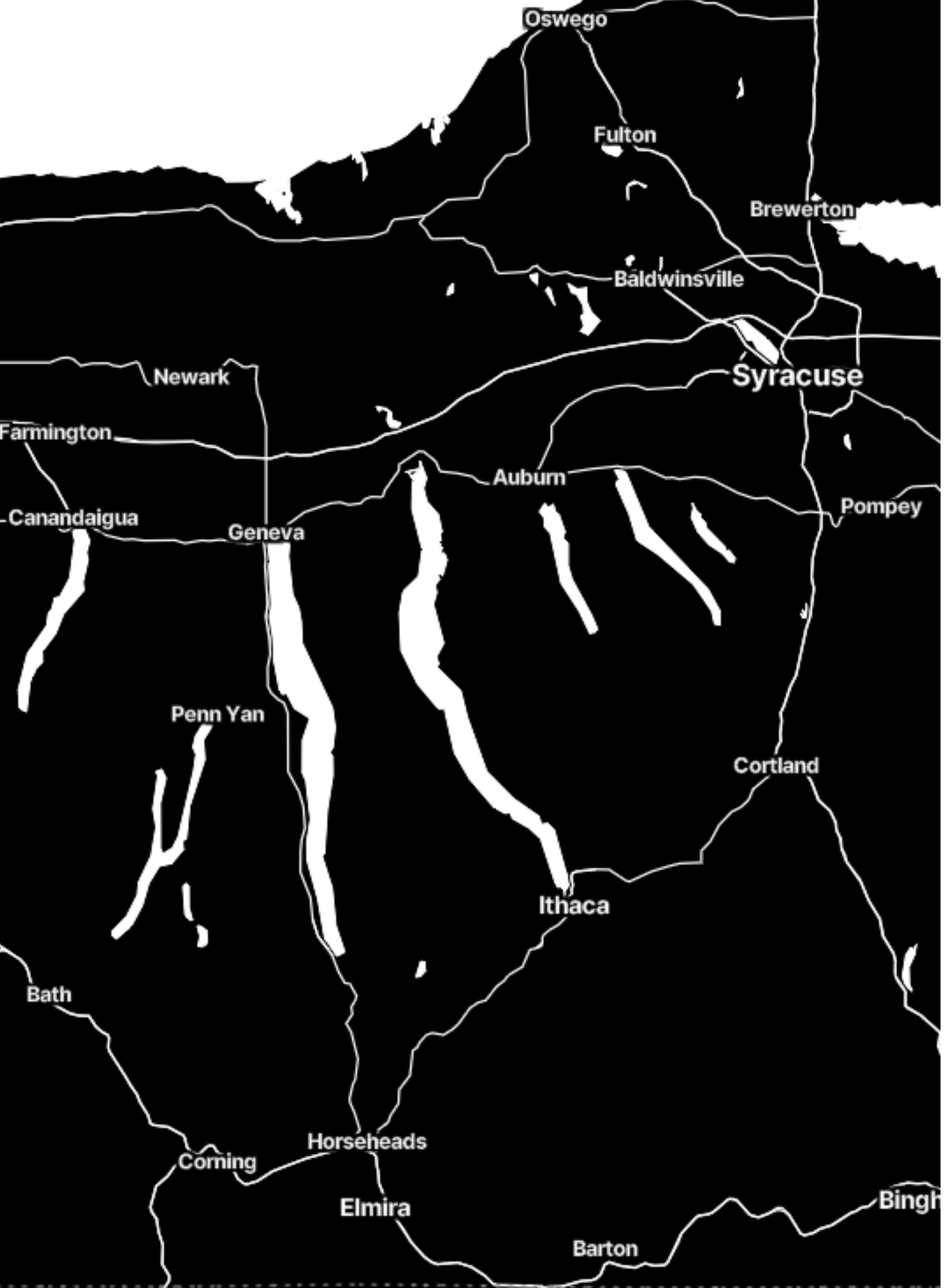

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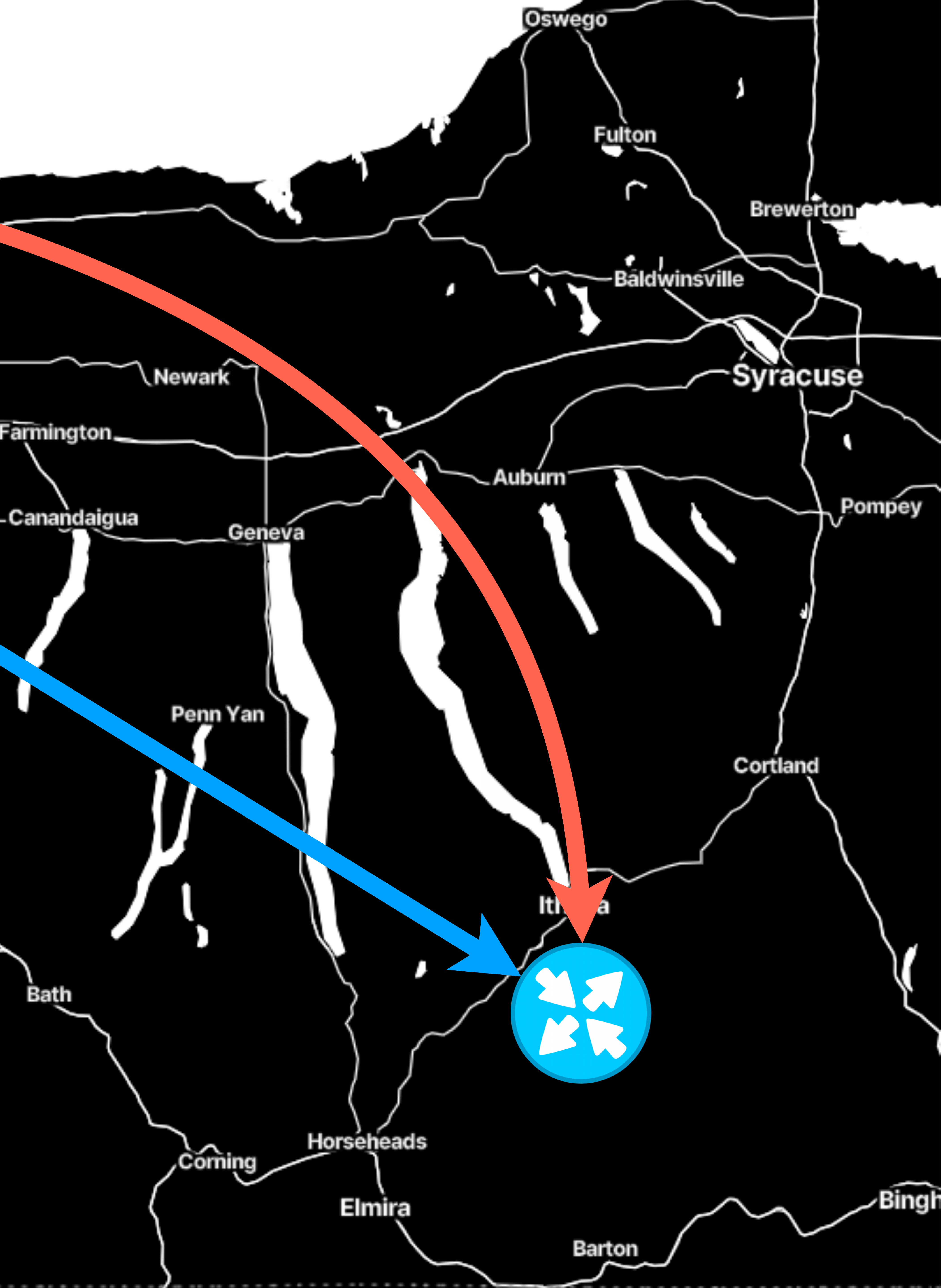

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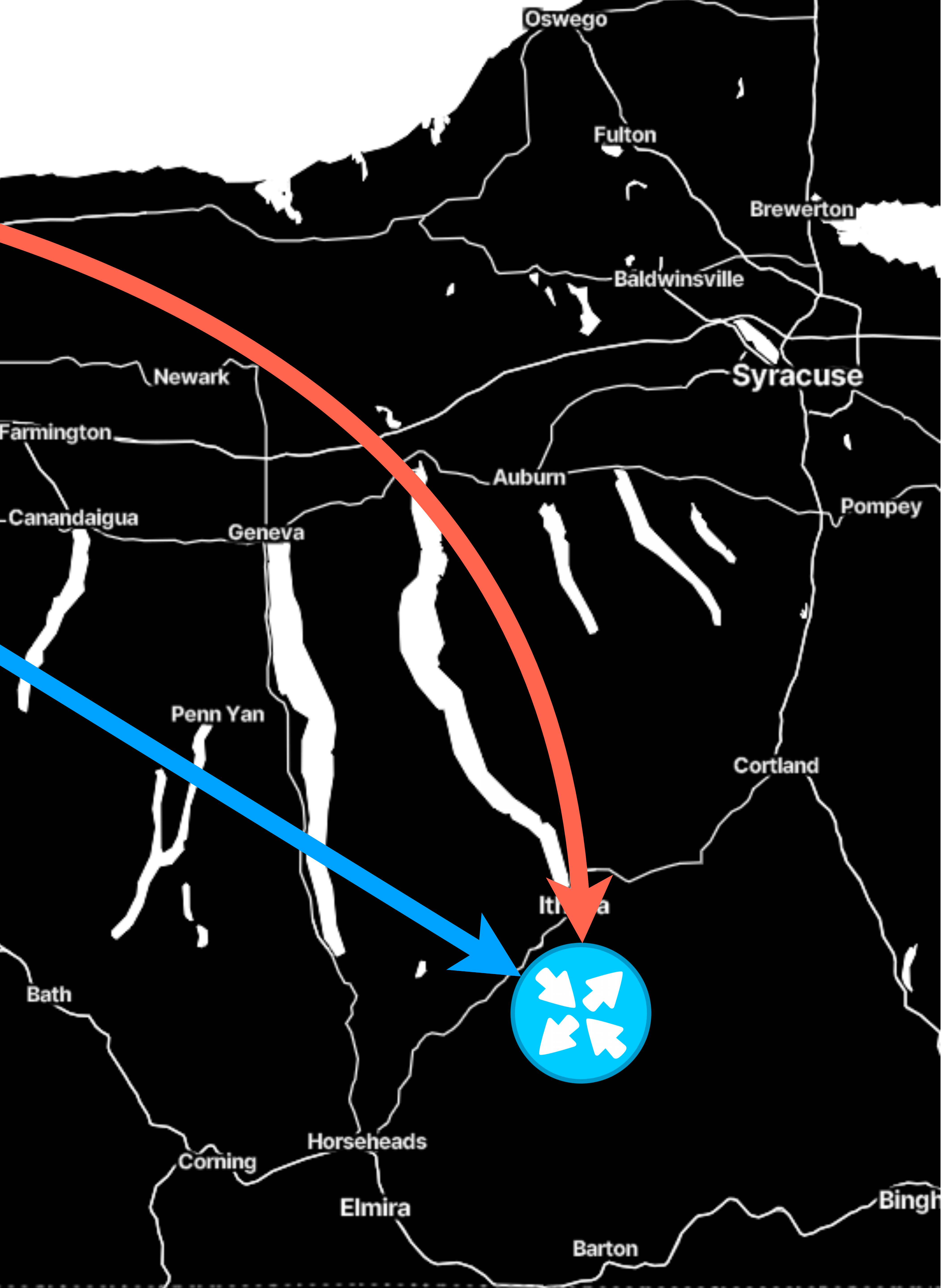

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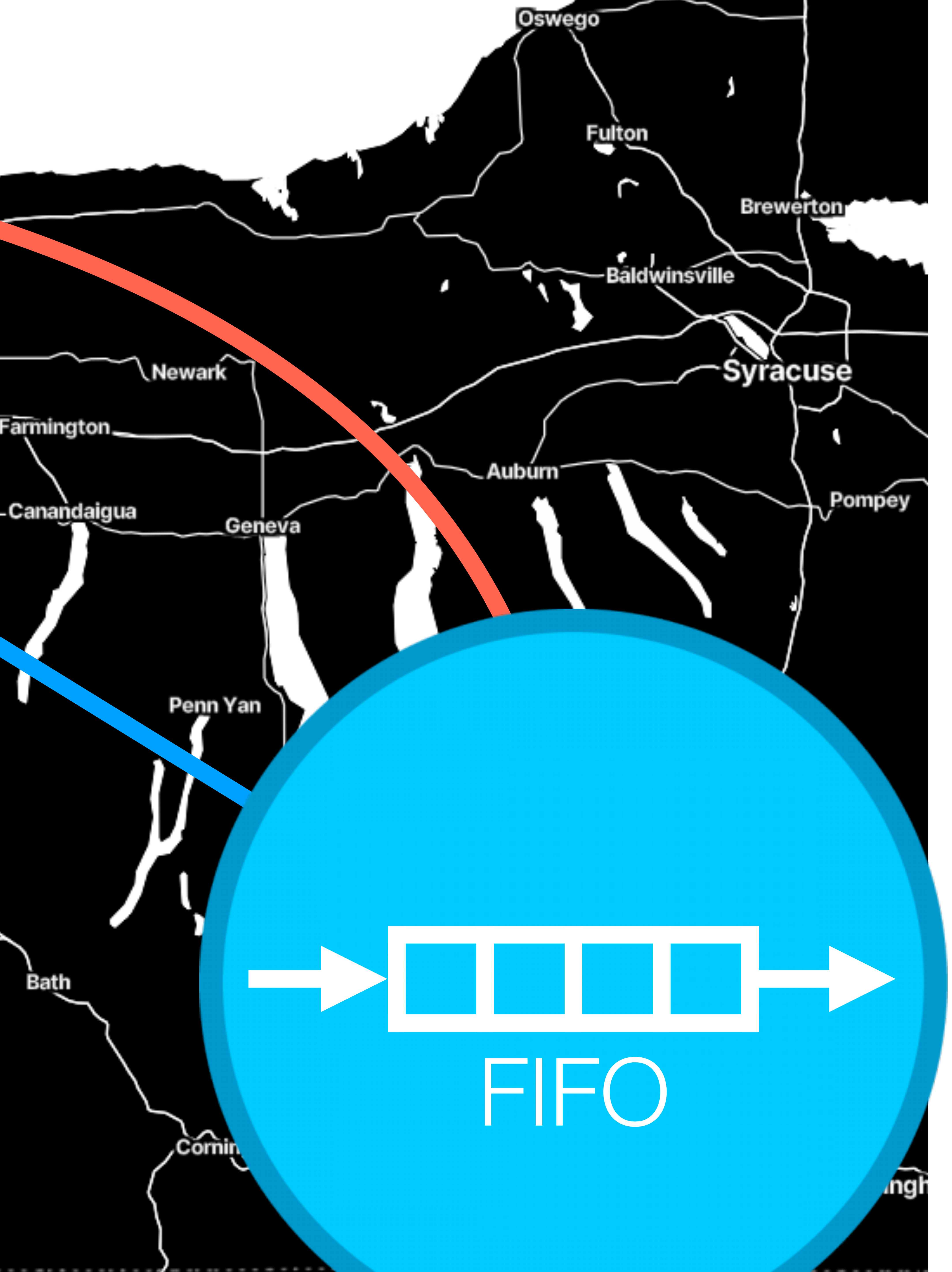

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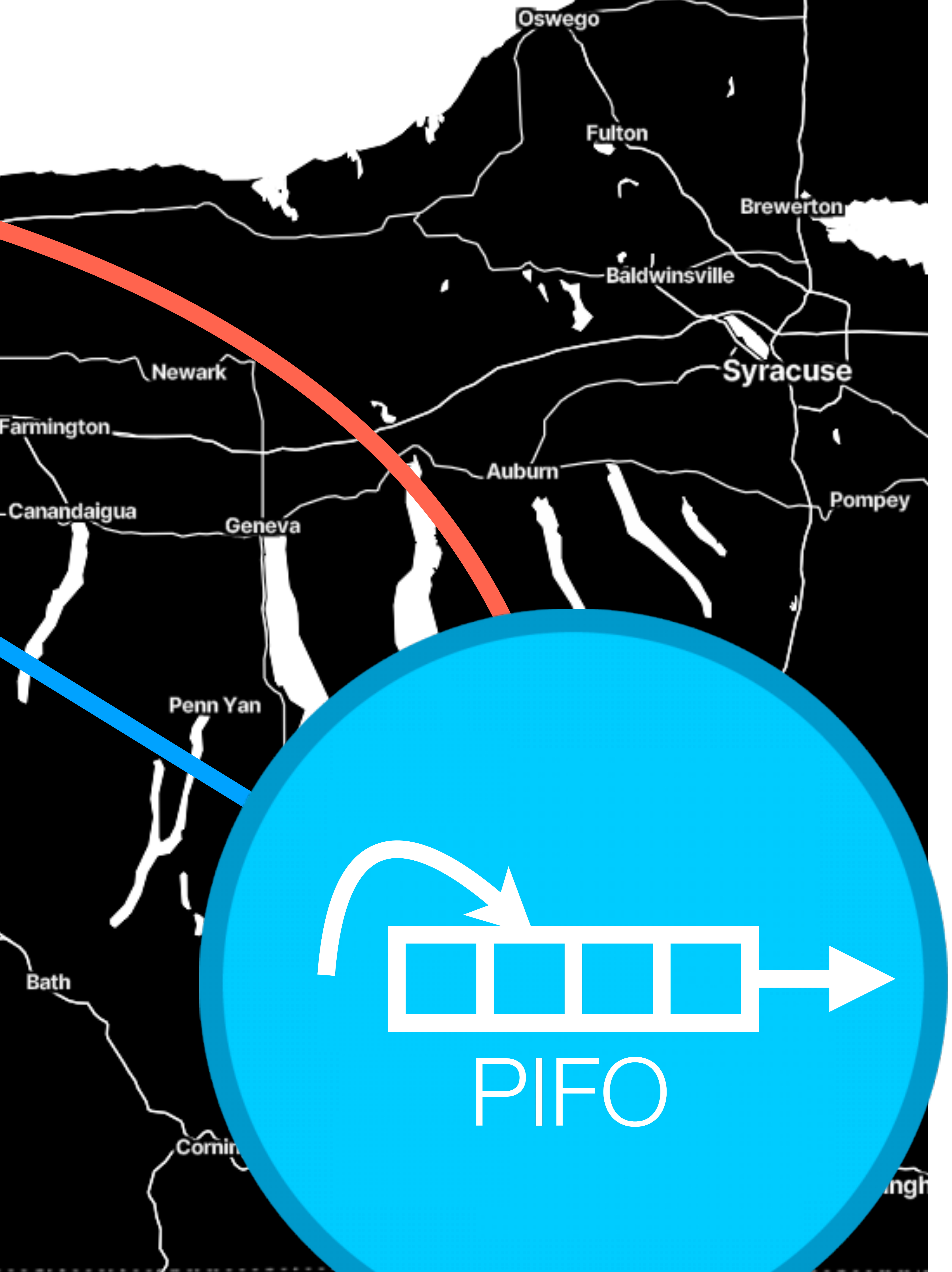

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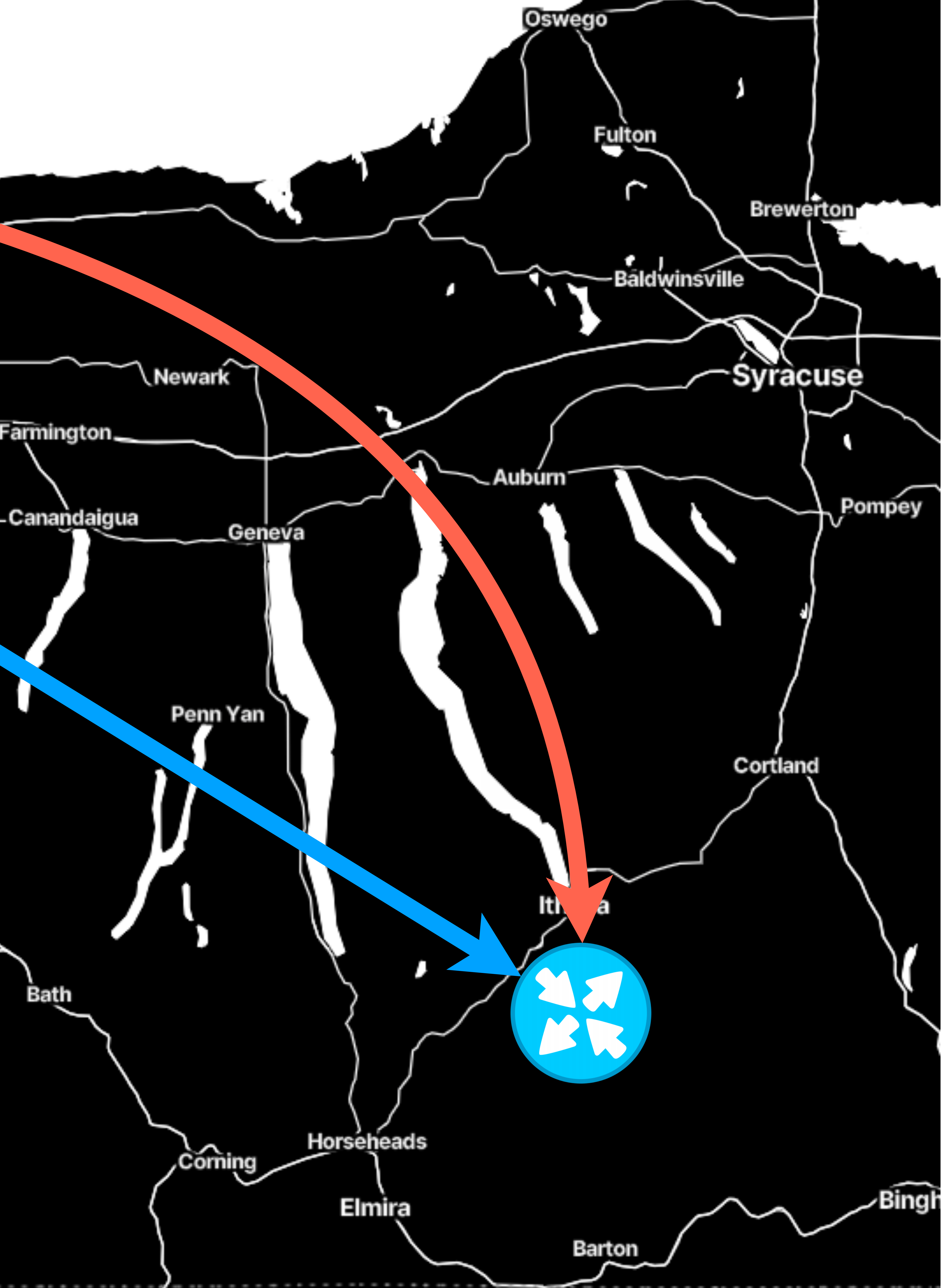

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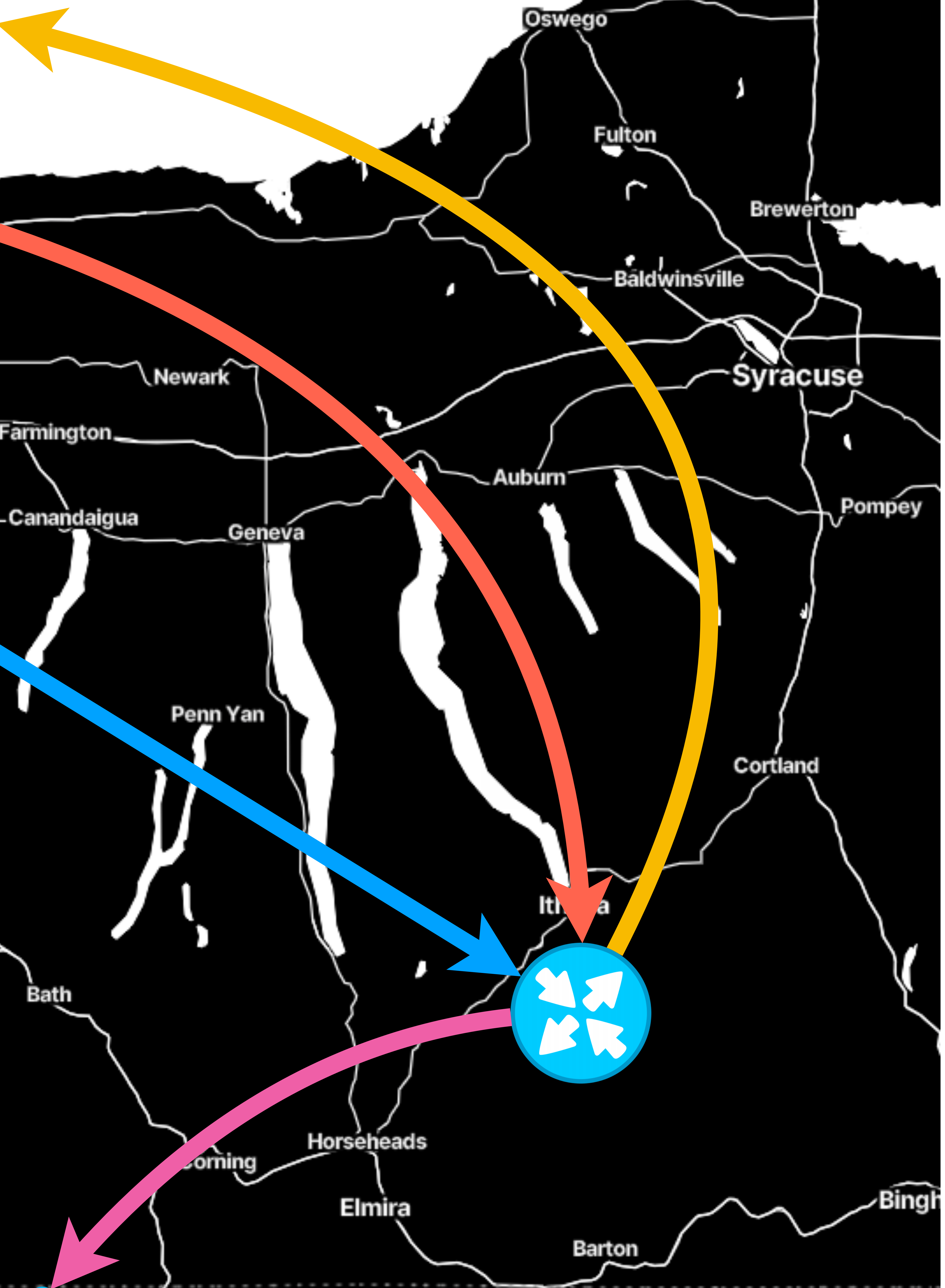

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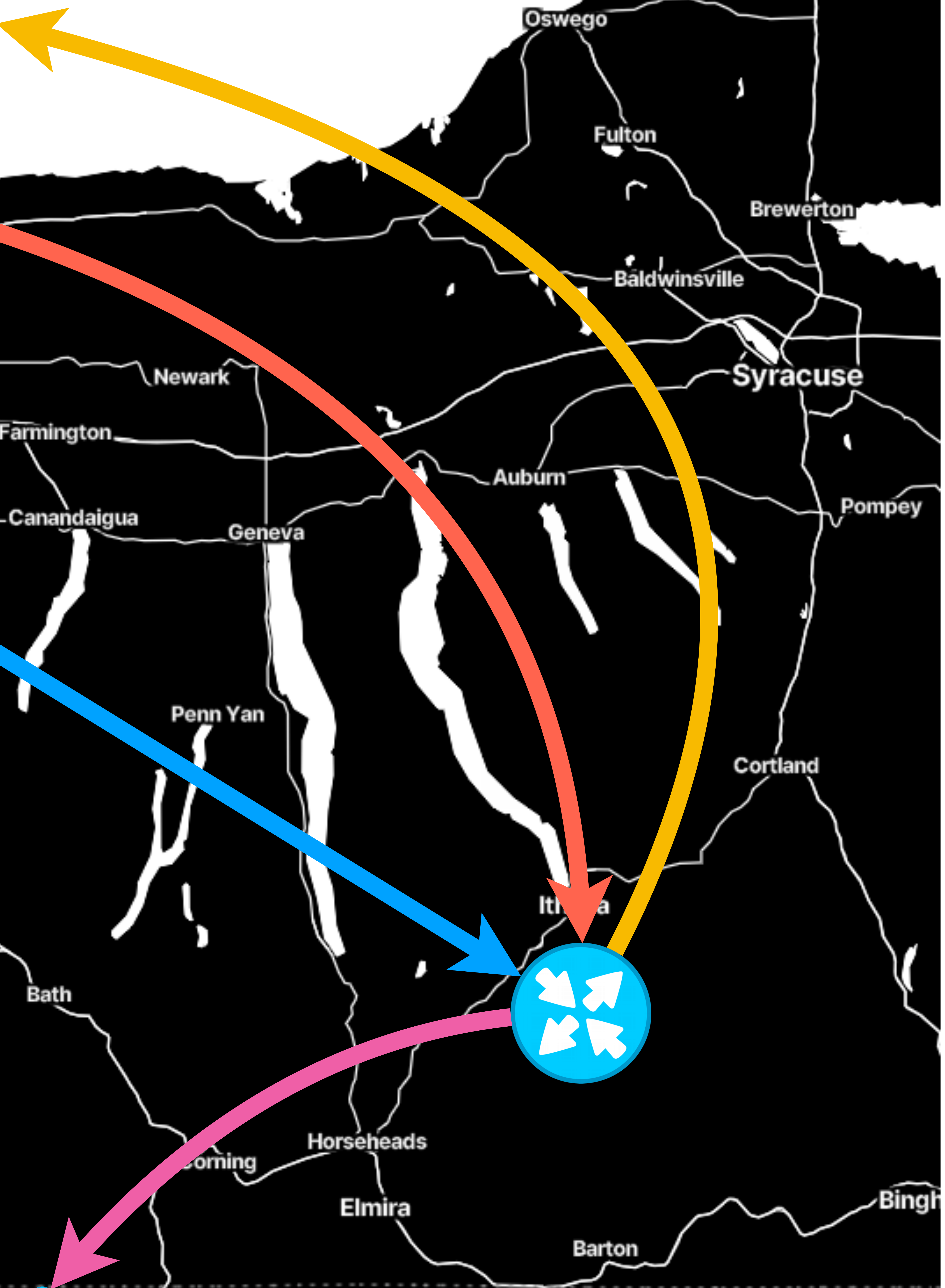

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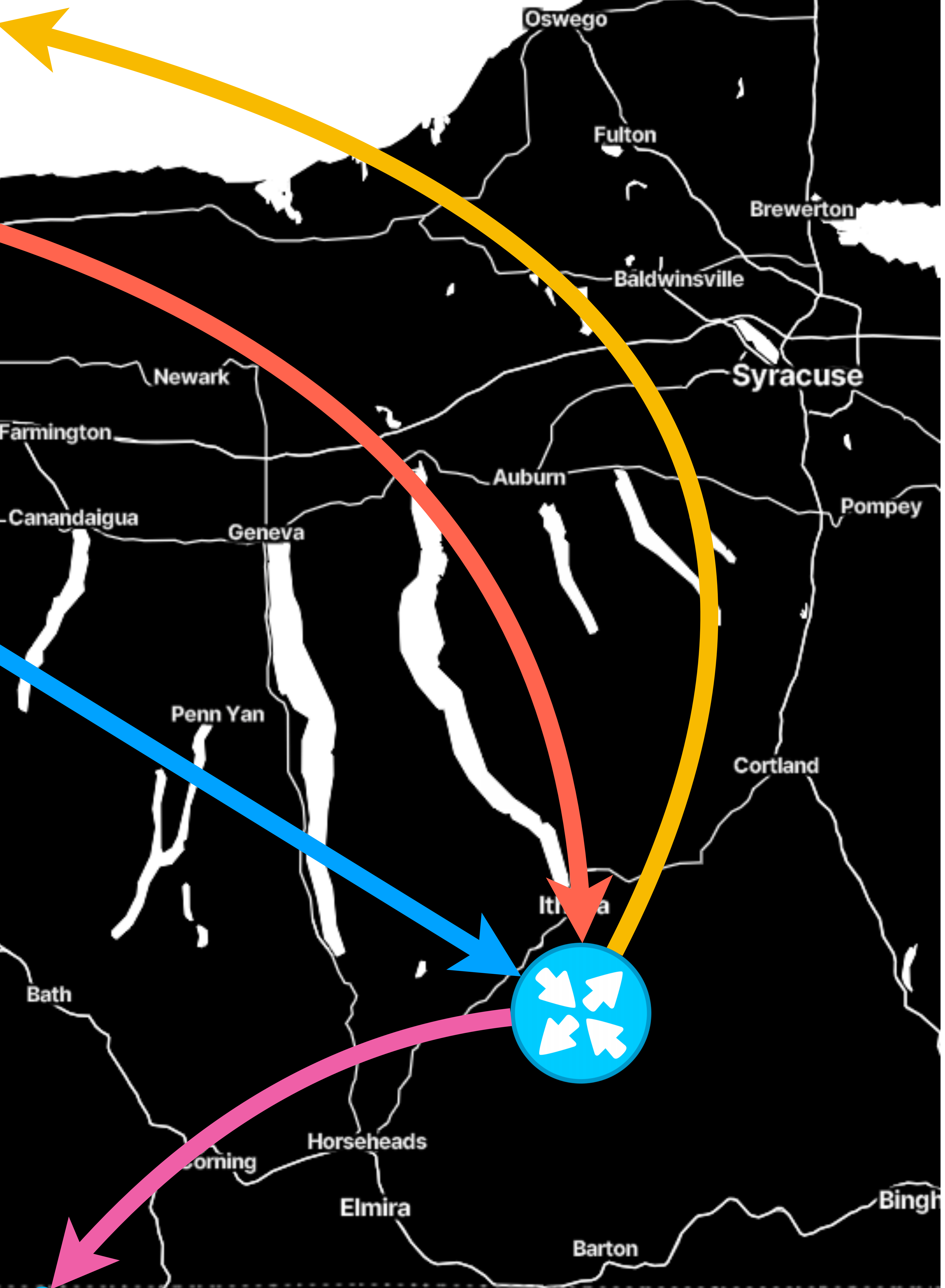

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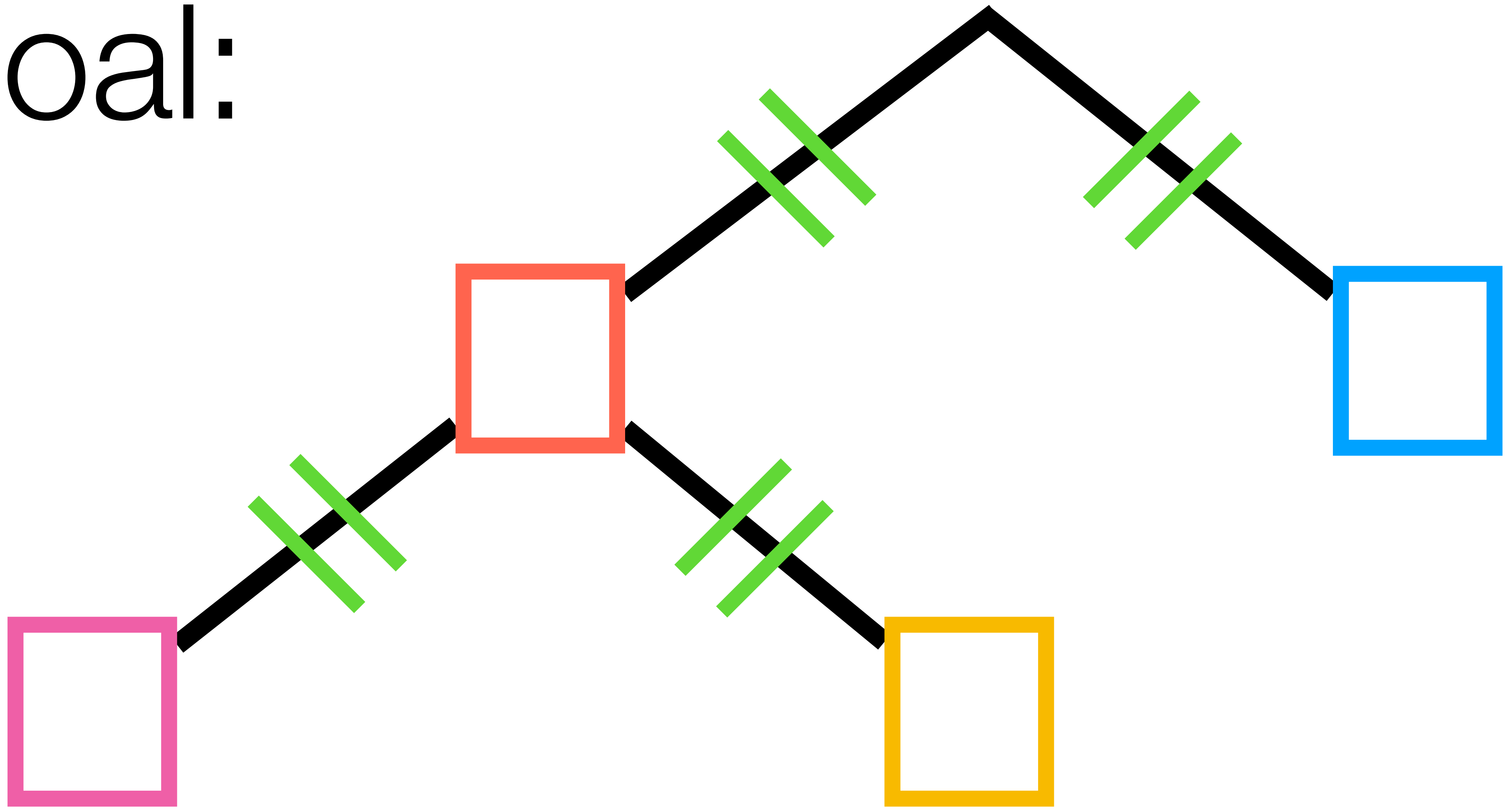

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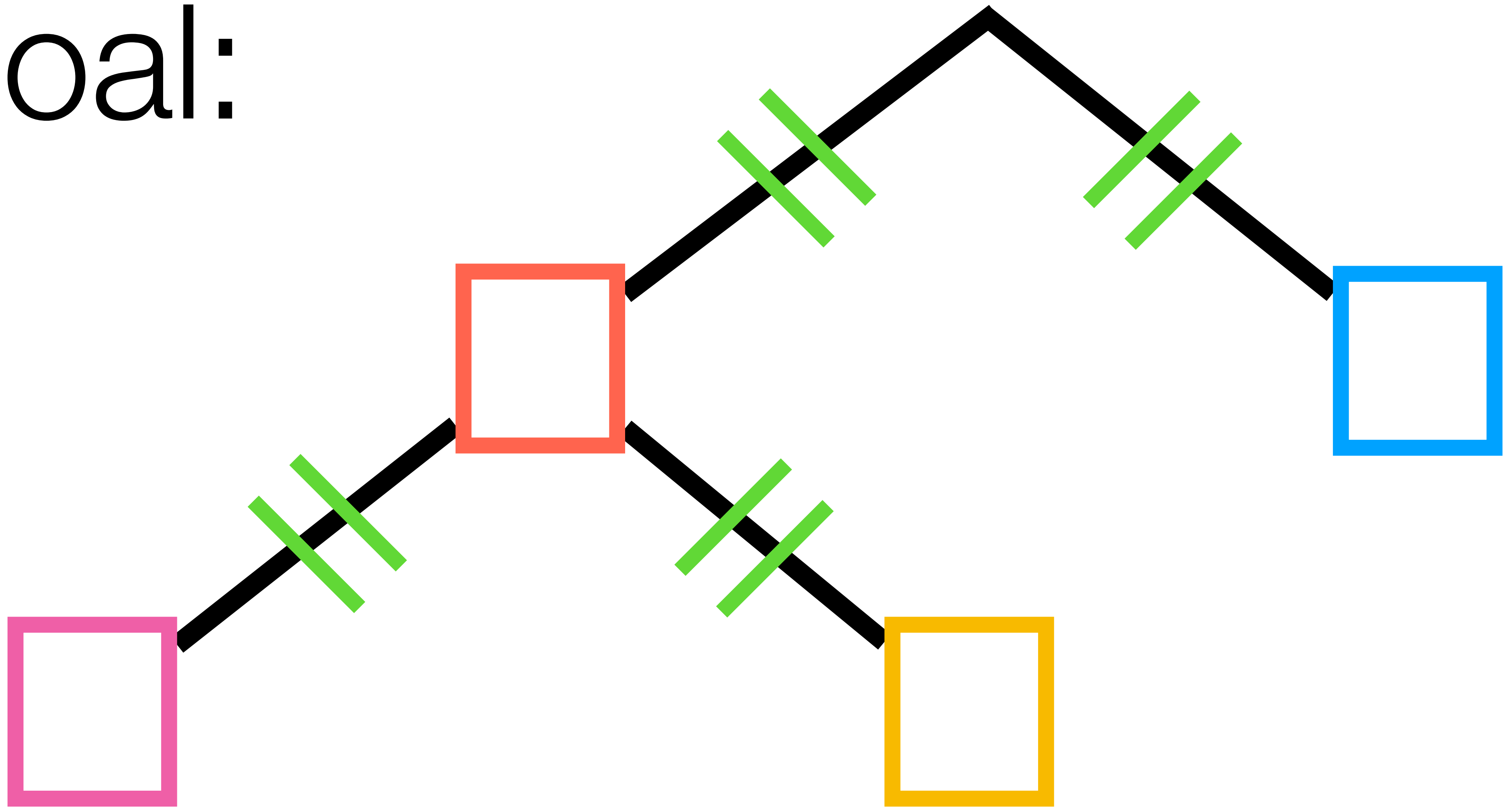

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PIFO Tree

New plan!

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Interleave
small, medium, and large
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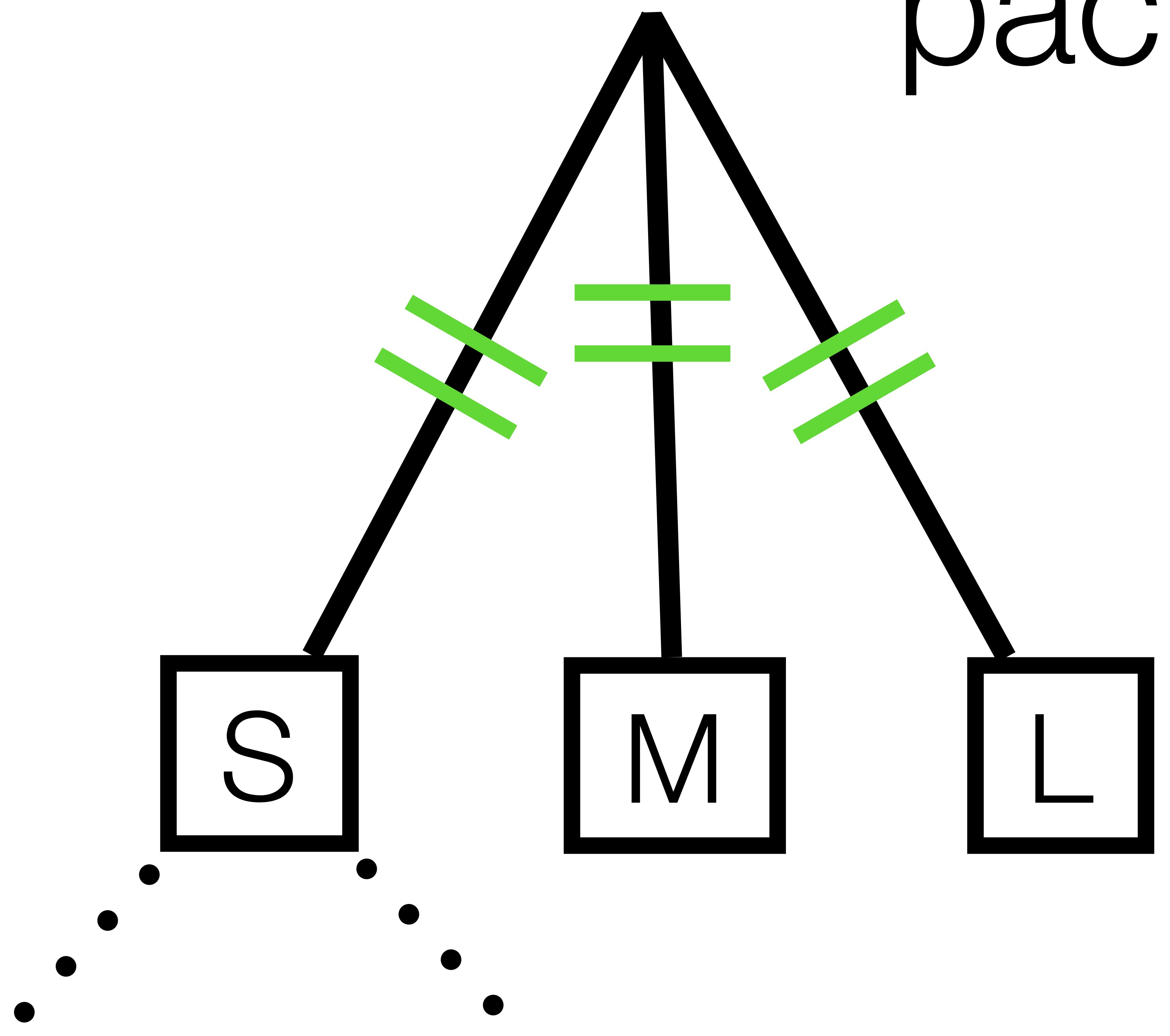

No general way to deploy our gadget.

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A human needs a *range* of trees.

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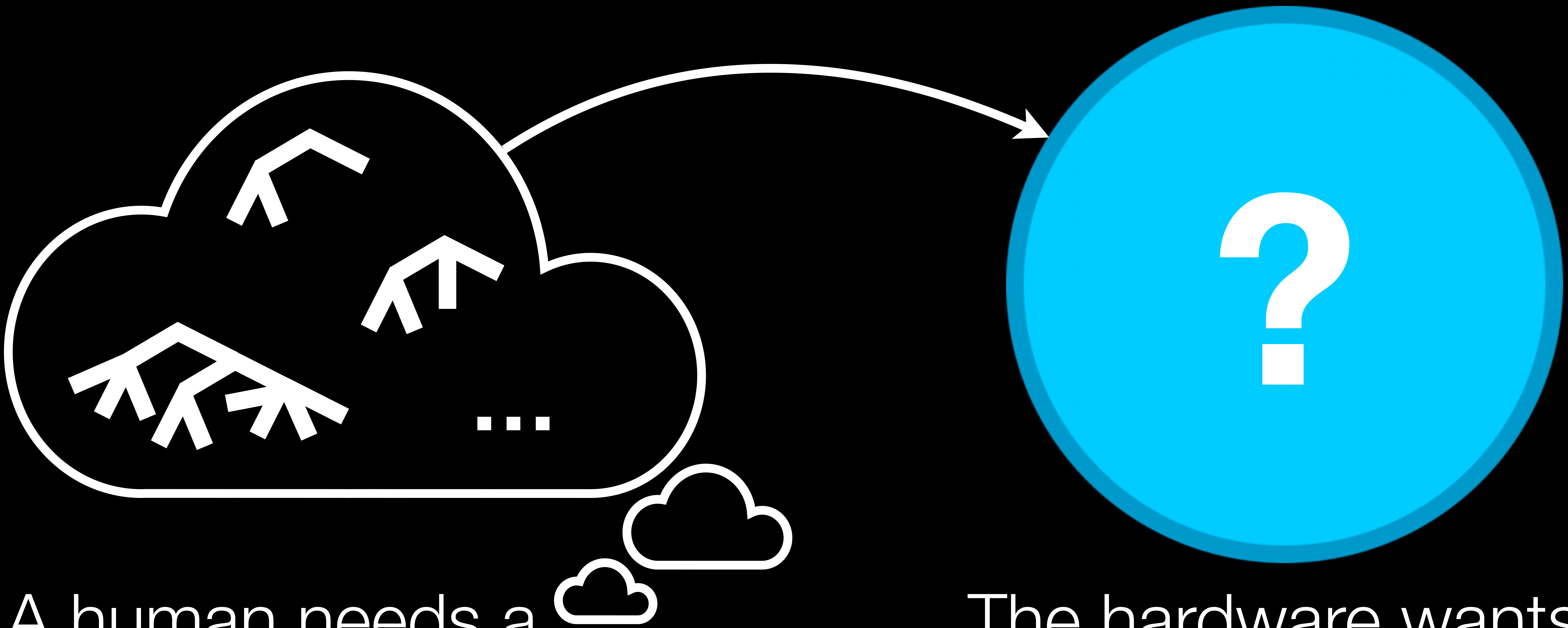

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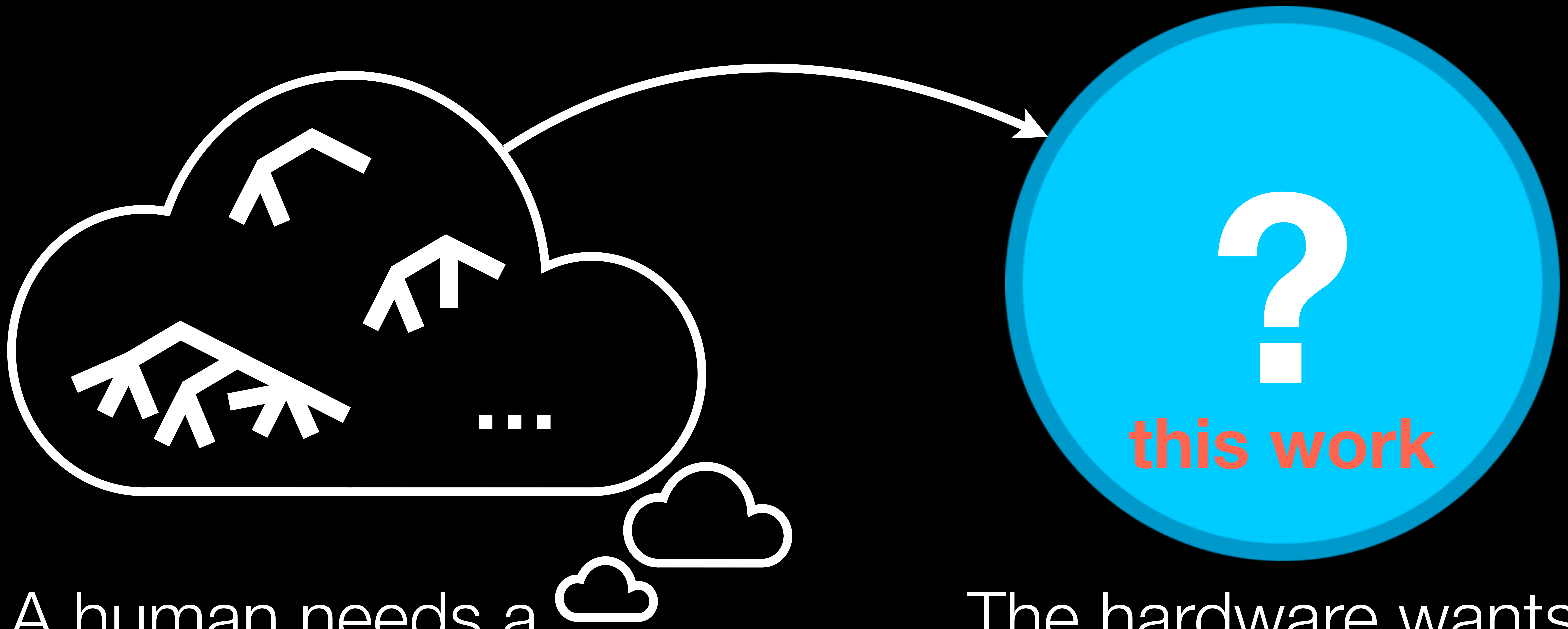

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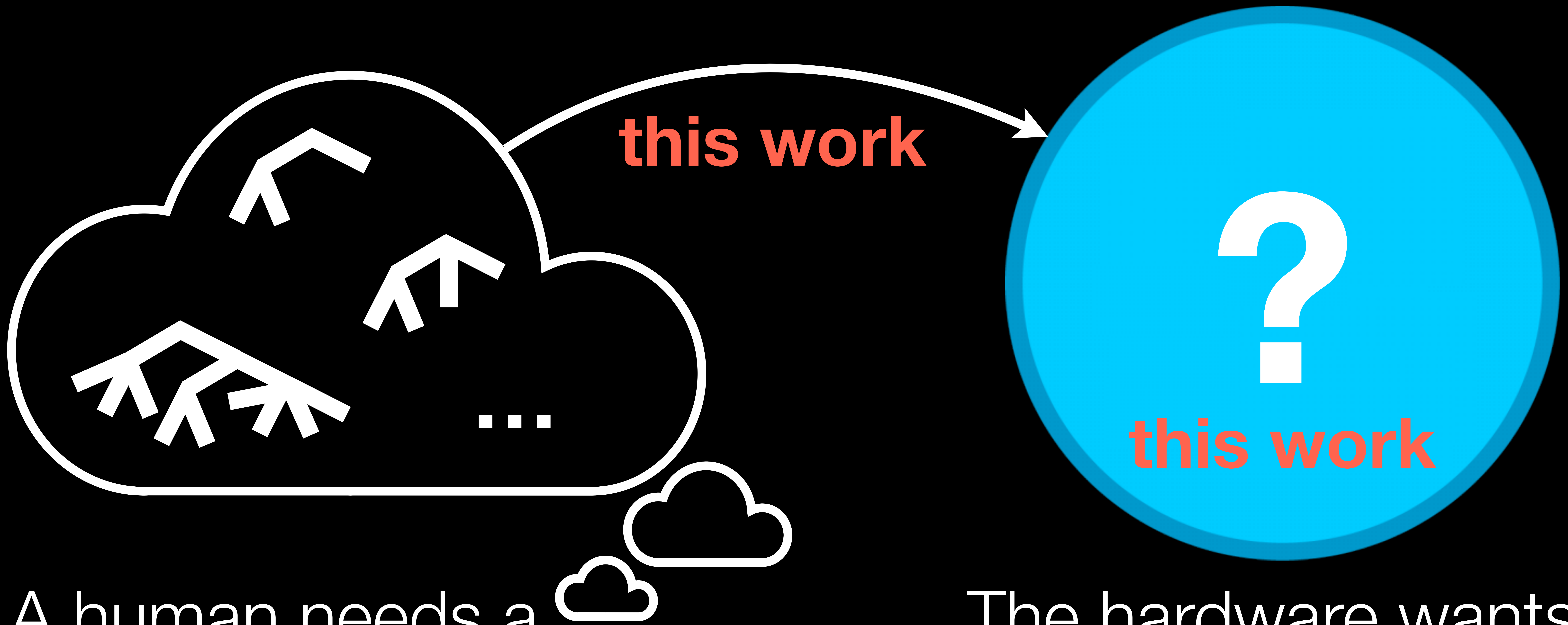

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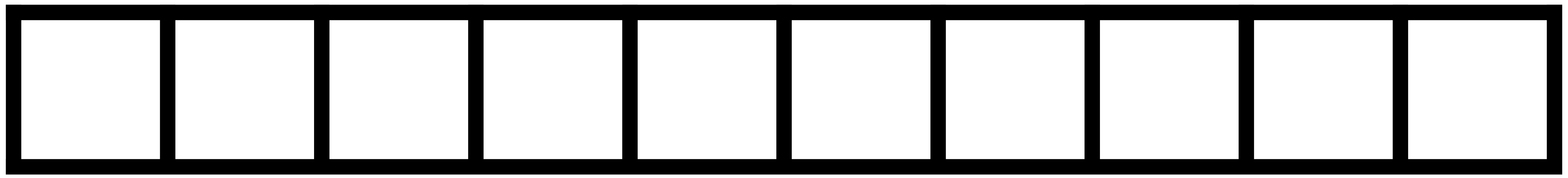
Aside: PIFO Trees

Sivaraman et al. at SIGCOMM ’16

Review: FIFO

Just an ordered collection.



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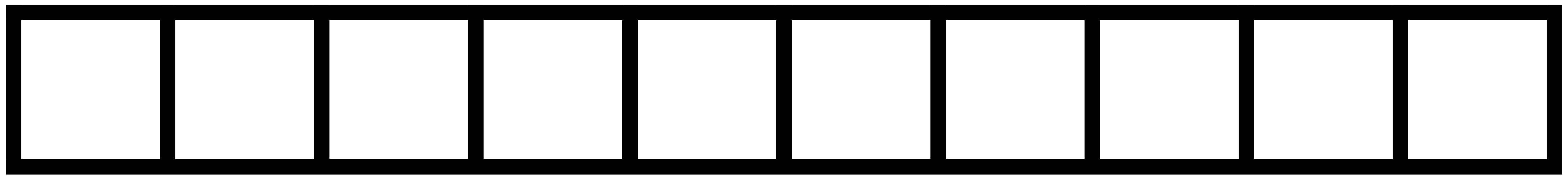

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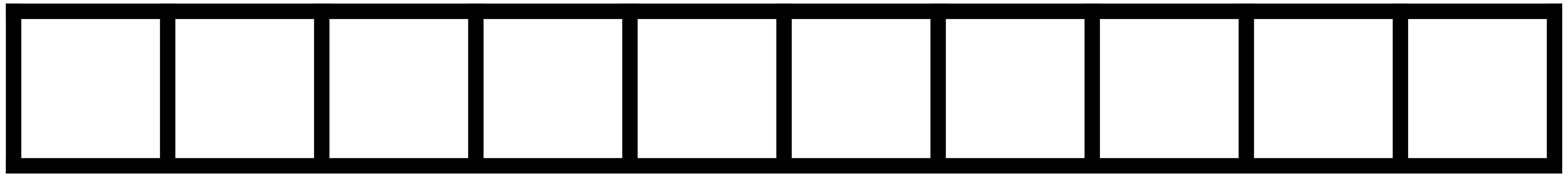

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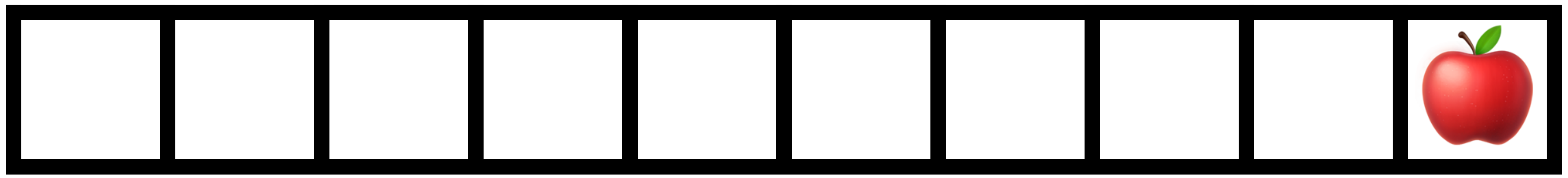

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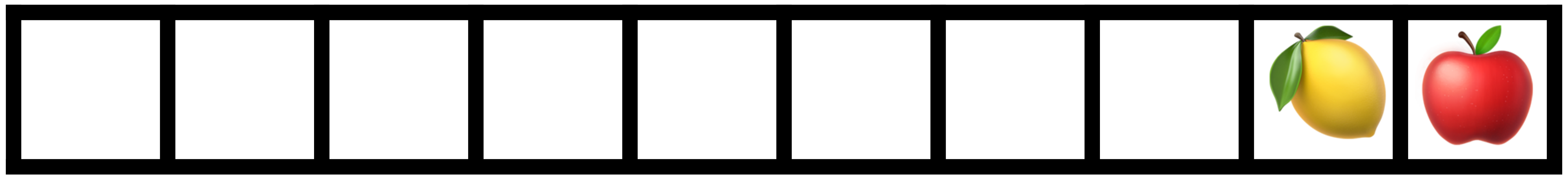

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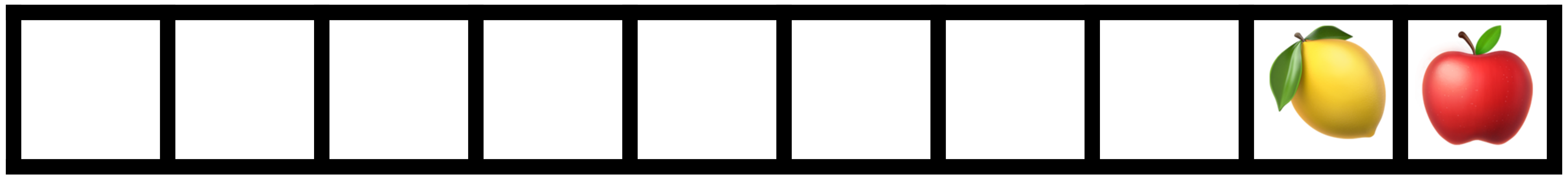

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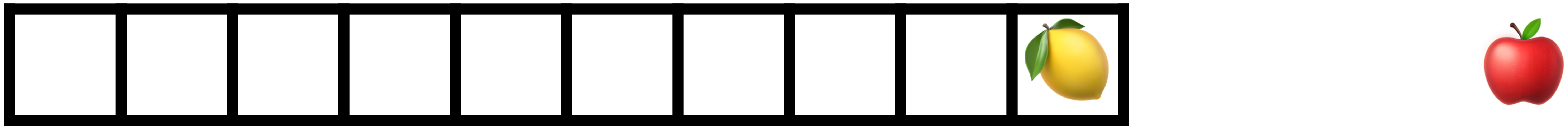

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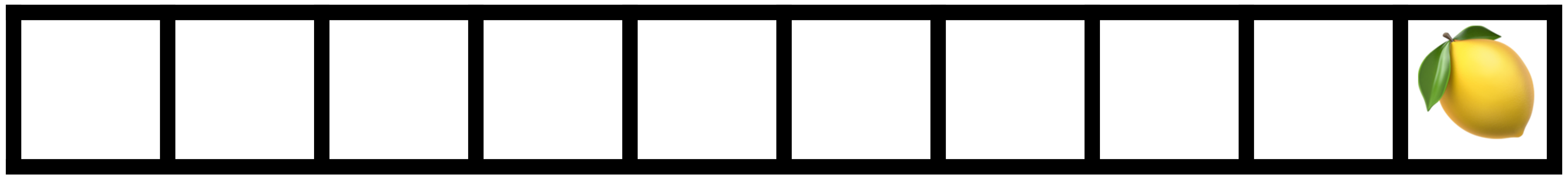

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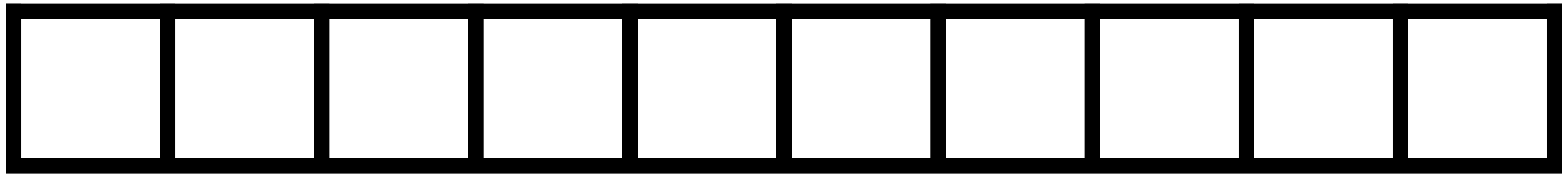

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Everything from before holds,
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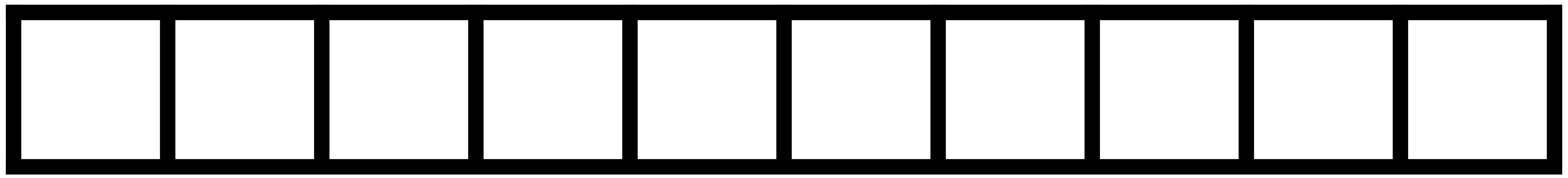

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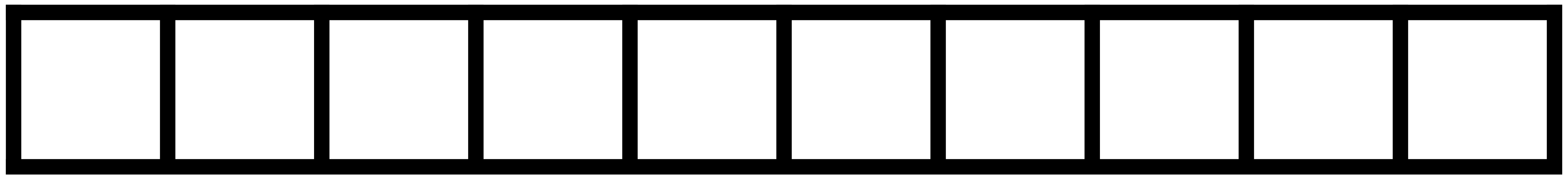

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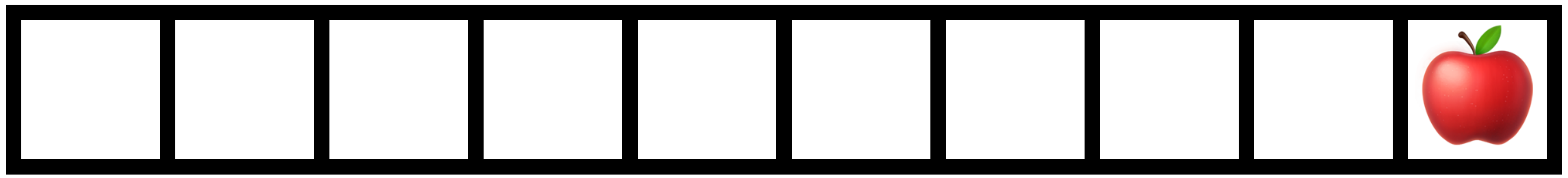

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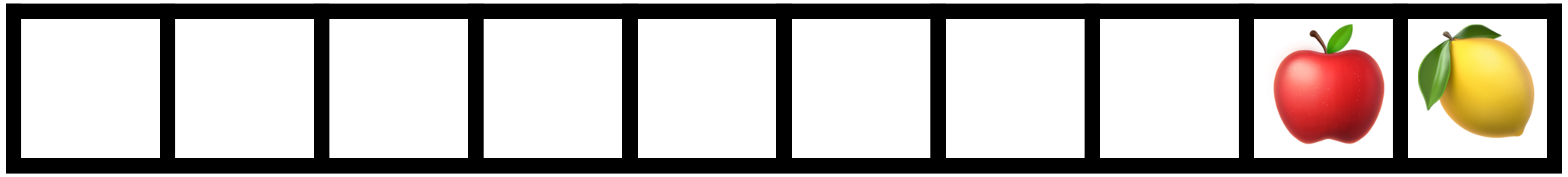

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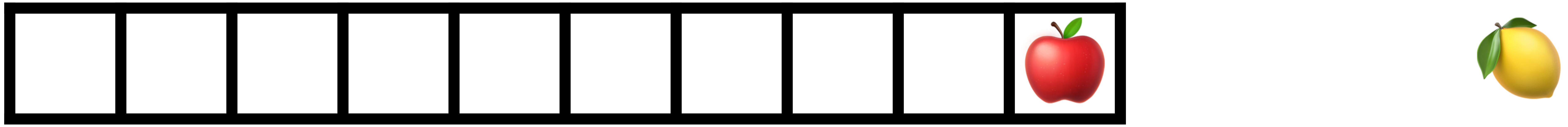

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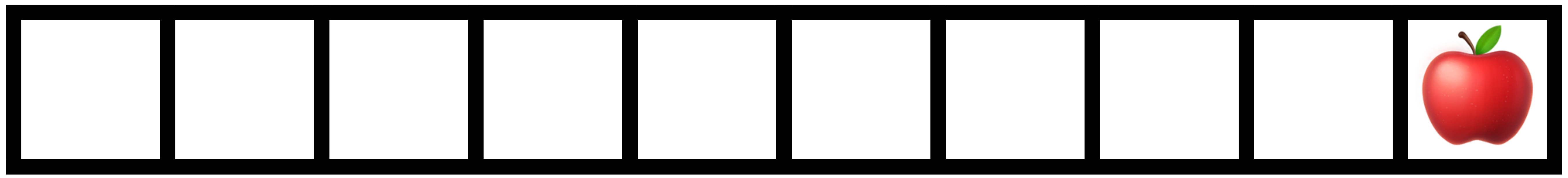

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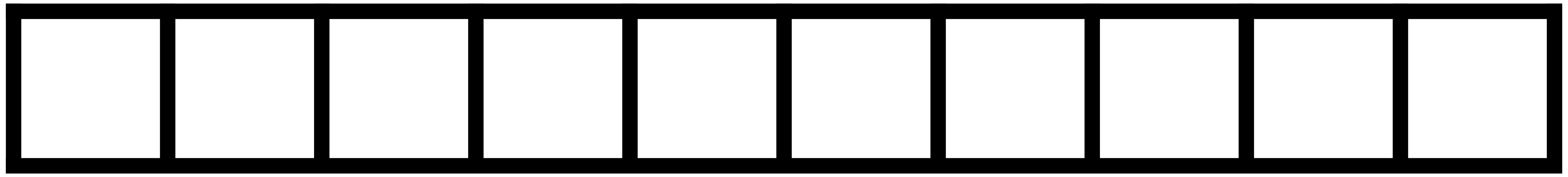

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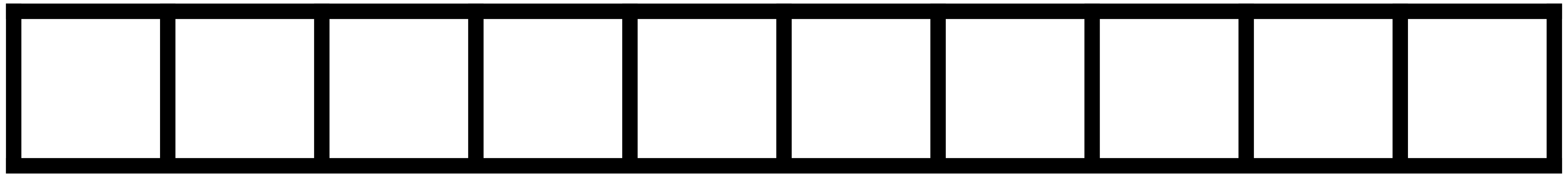

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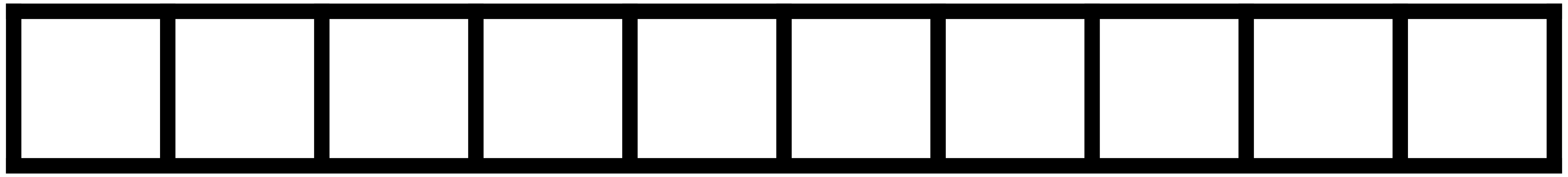

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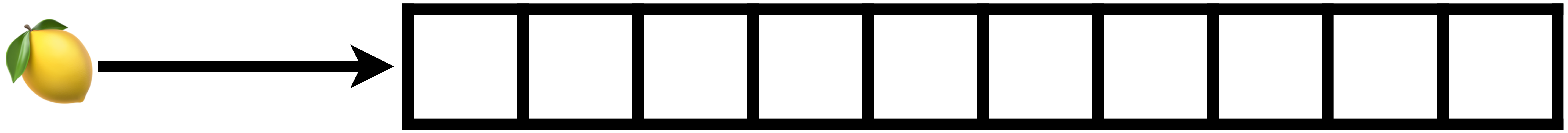

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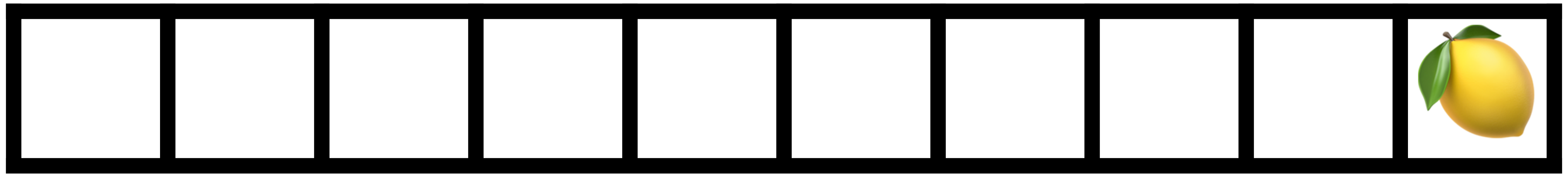

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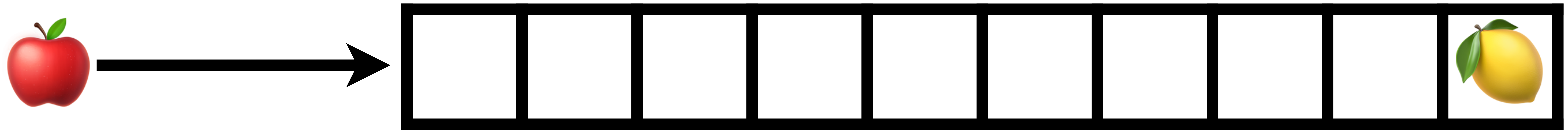

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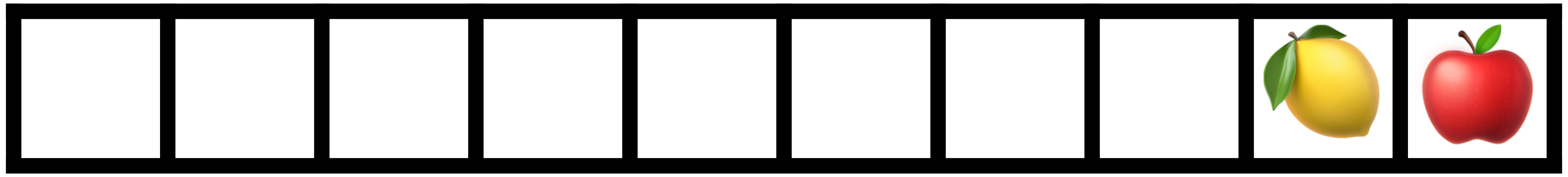

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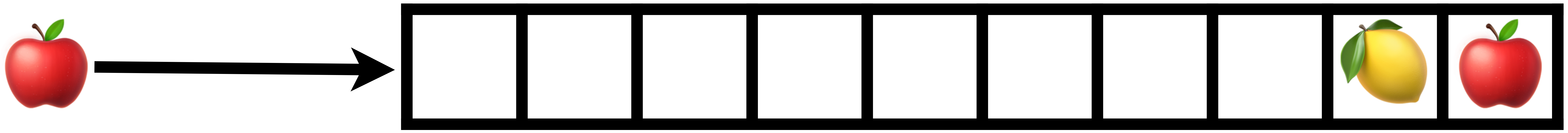

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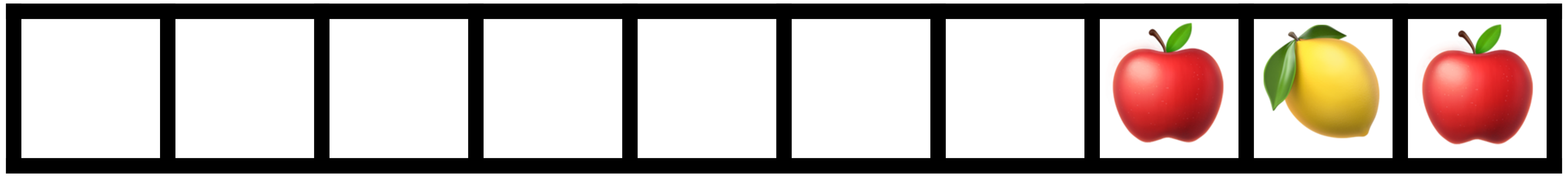
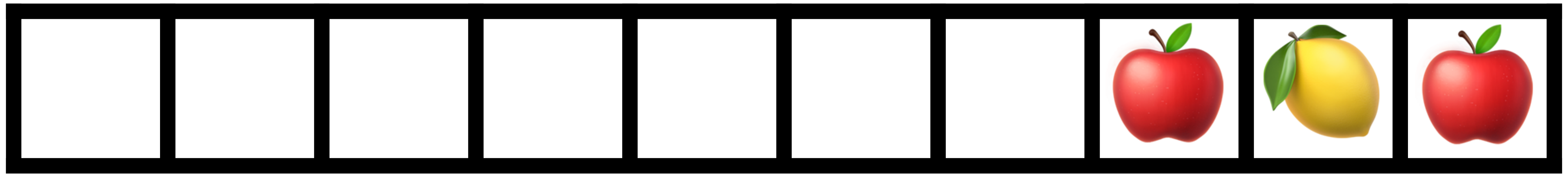
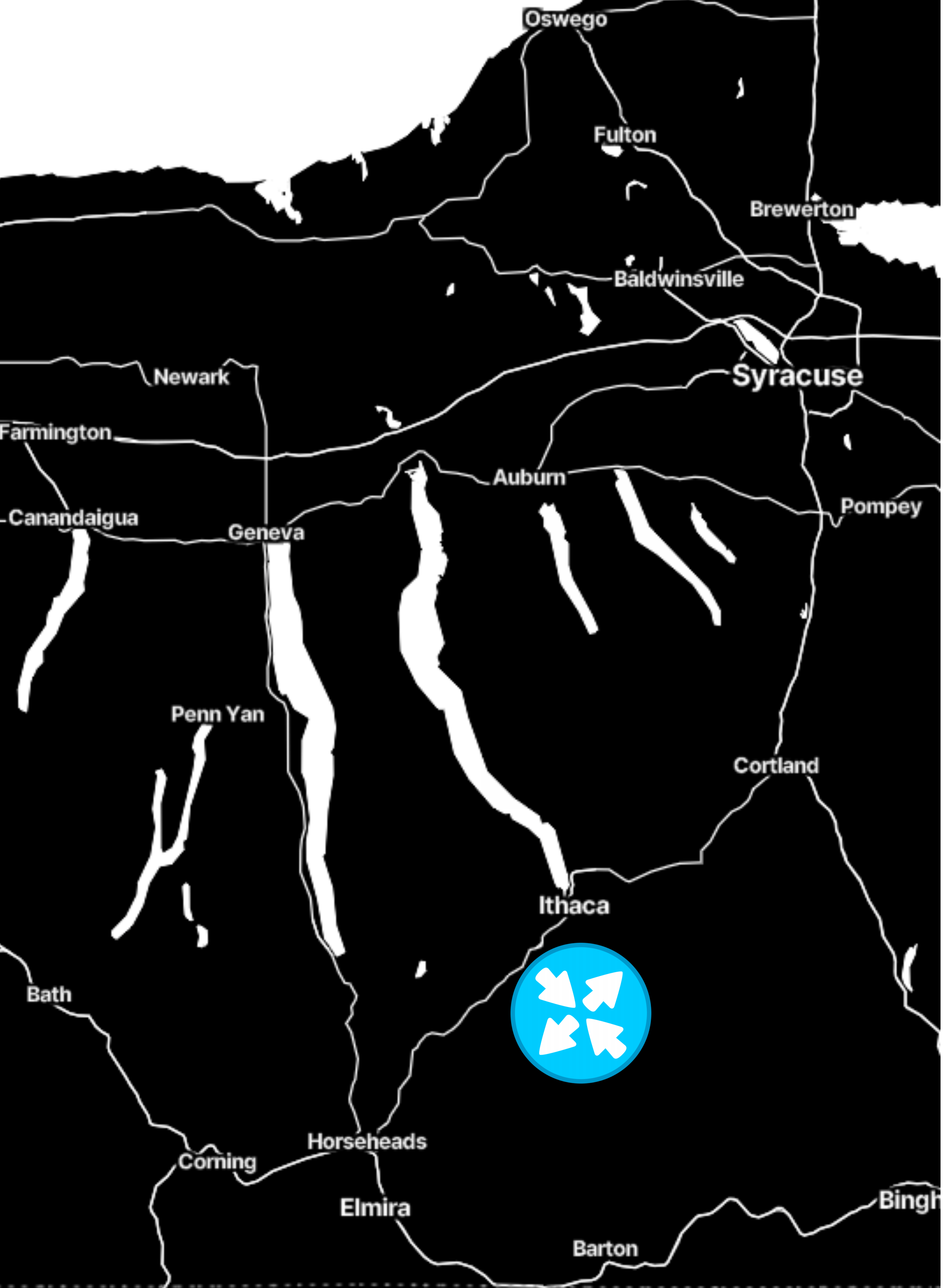


Introducing: PIFO

Just a PQ, with a ranking function,
but with *rank-ties* broken in FIFO order.

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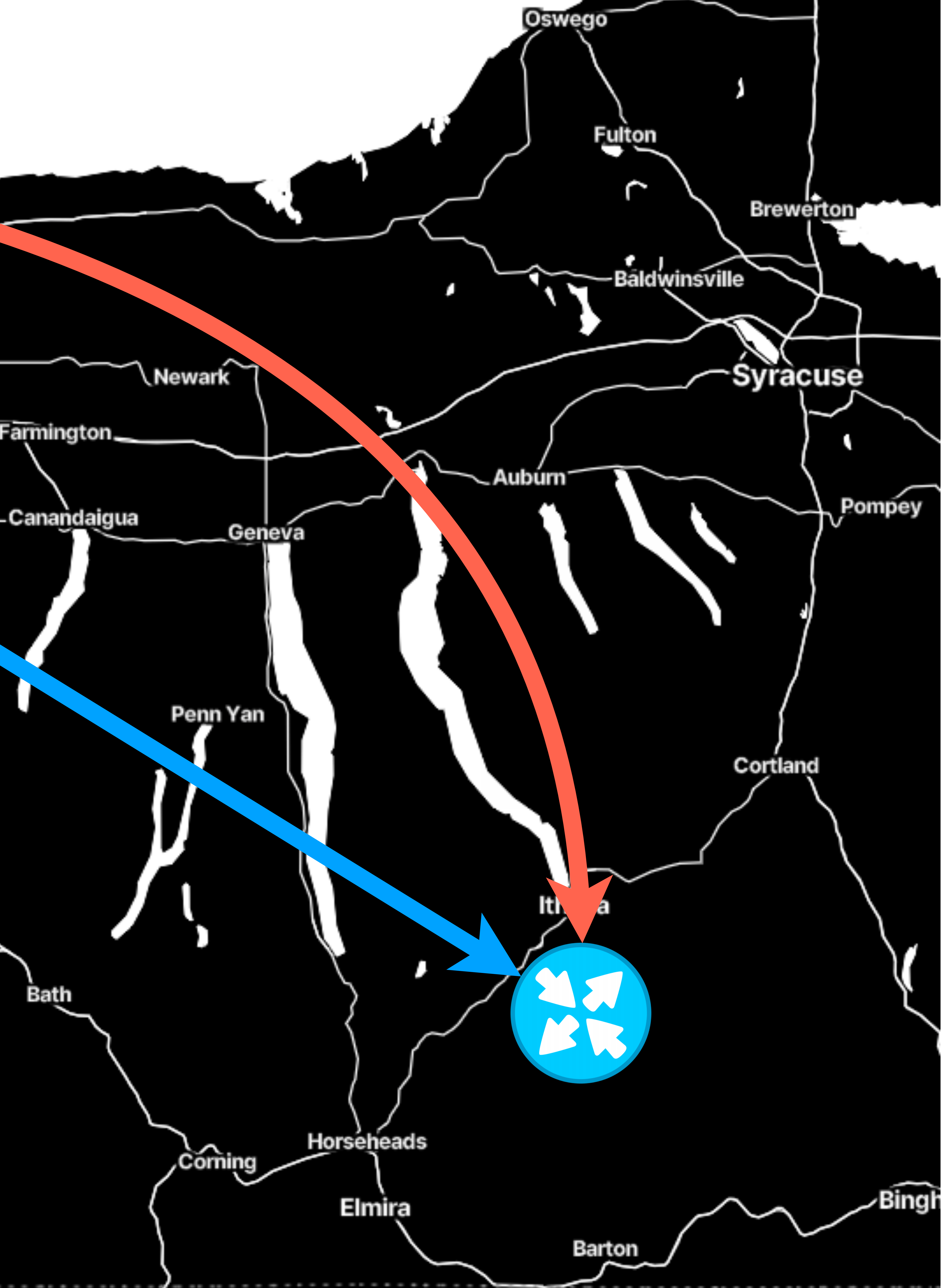

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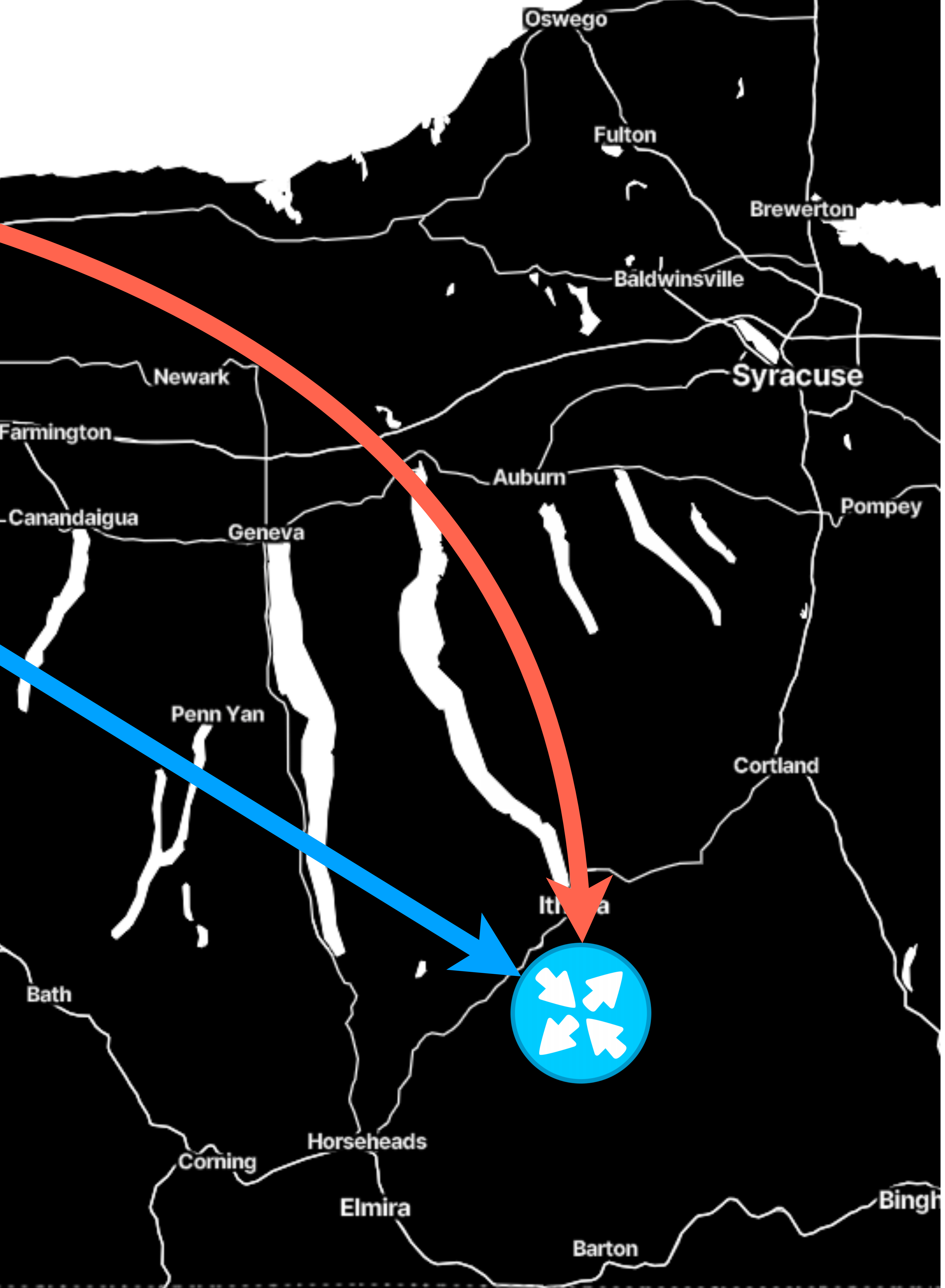

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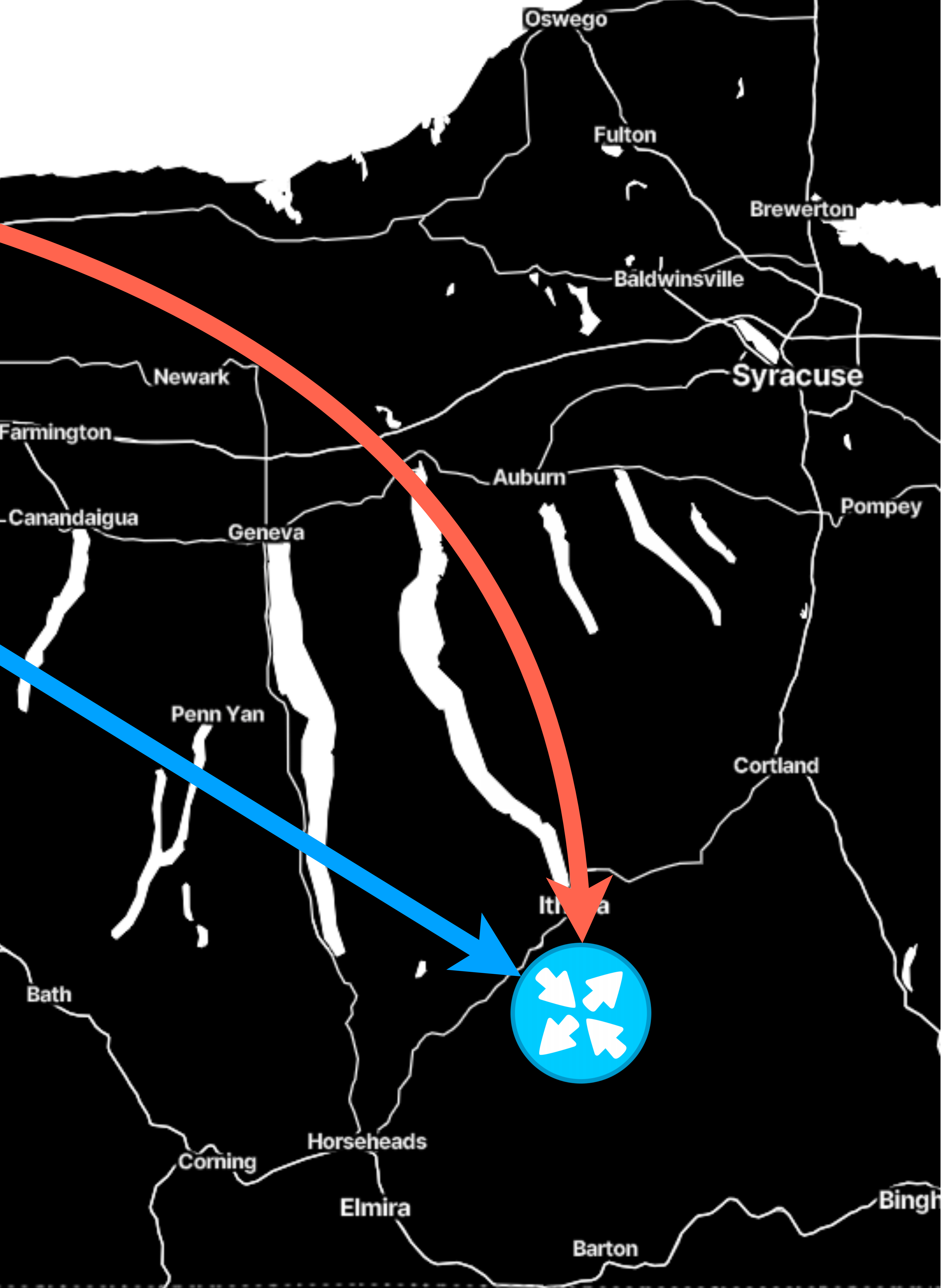

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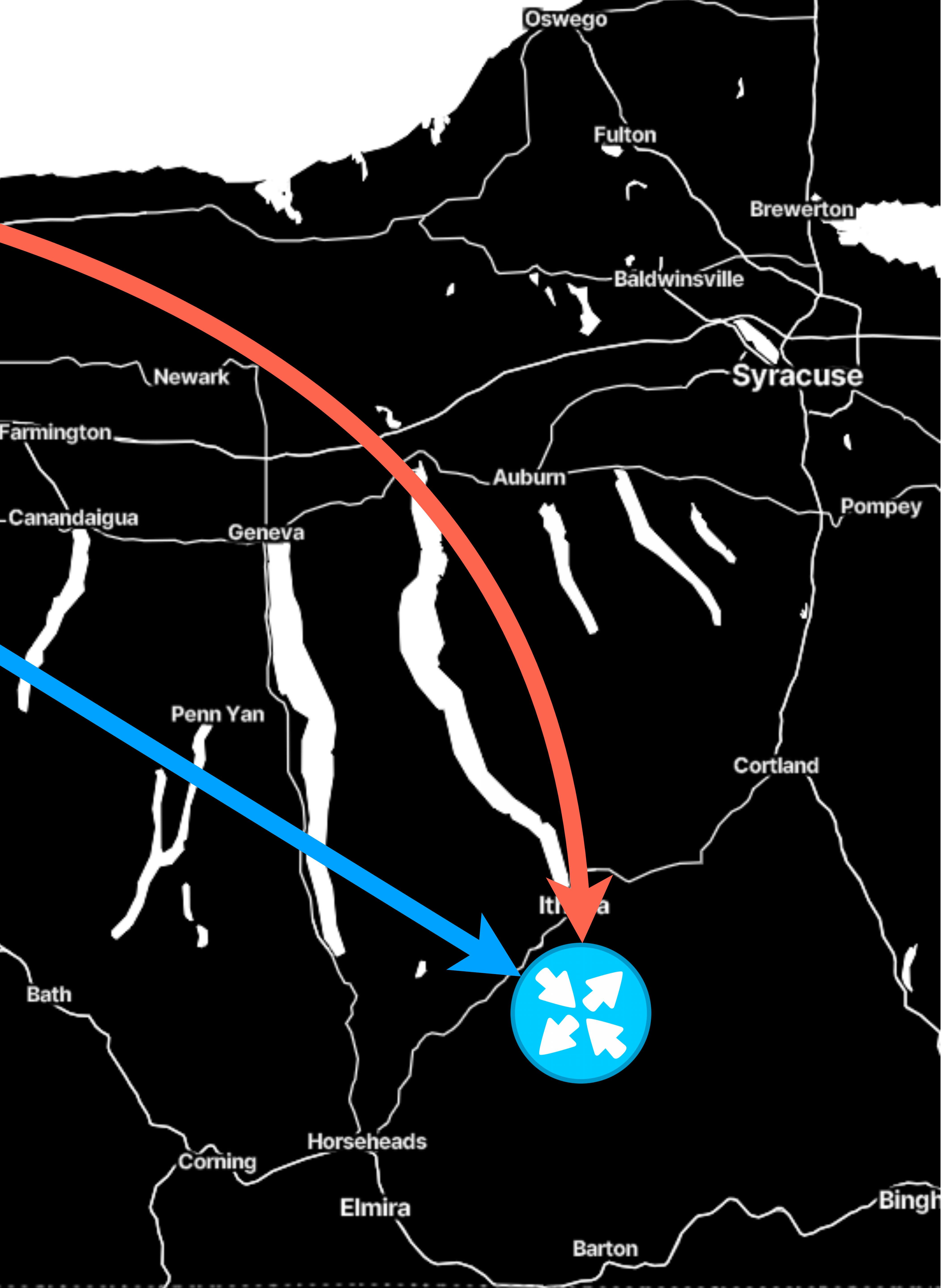

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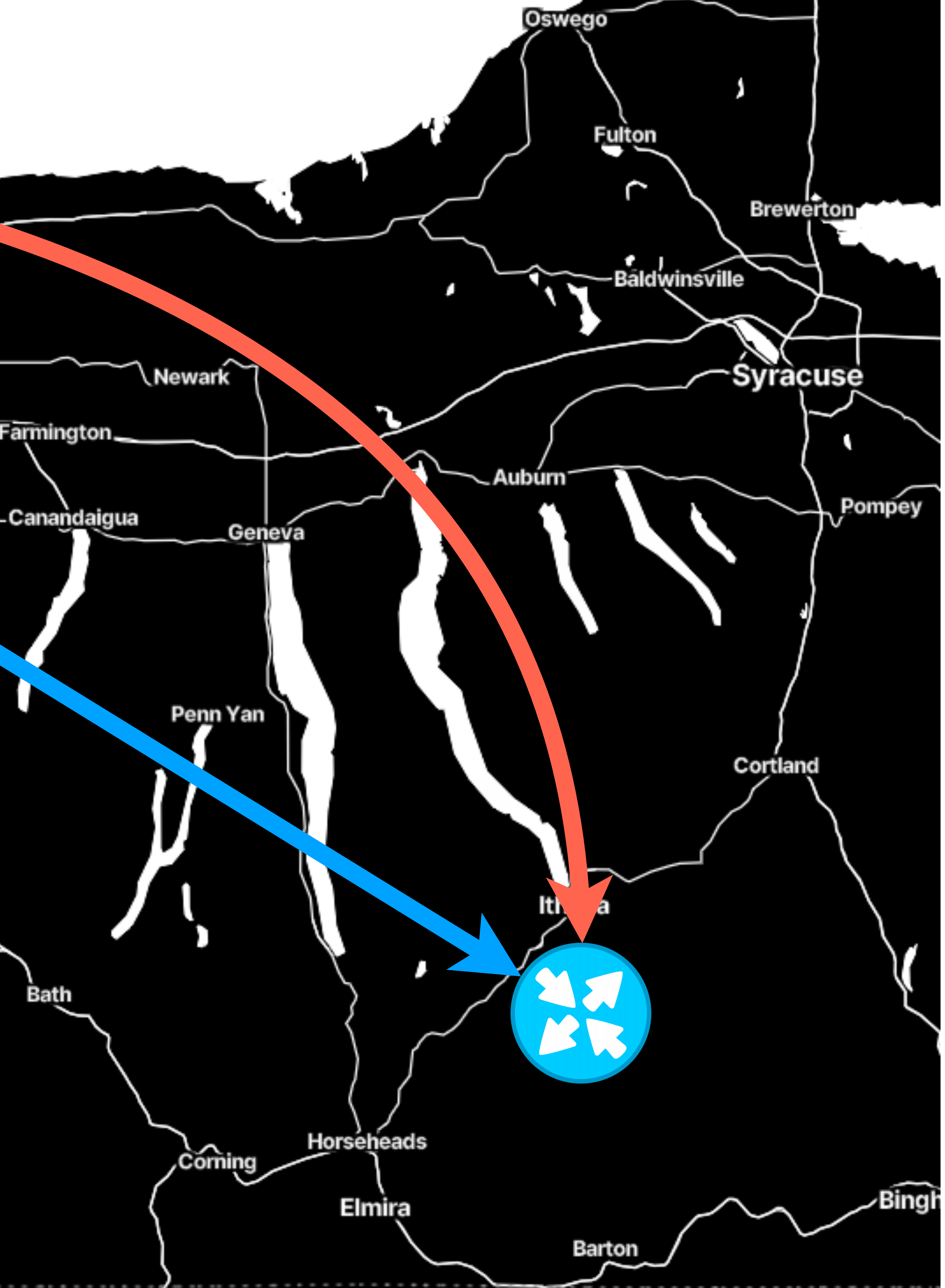

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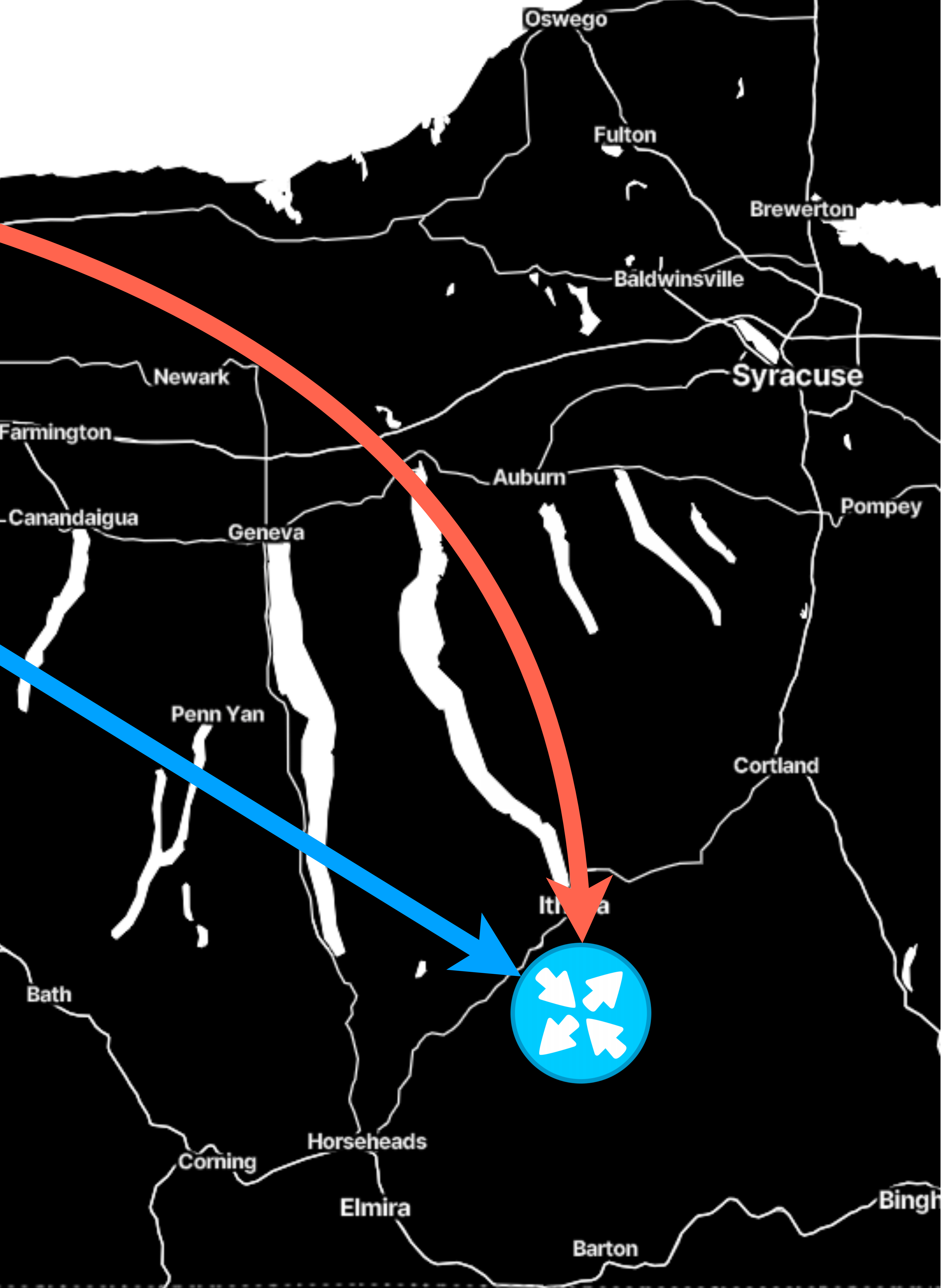

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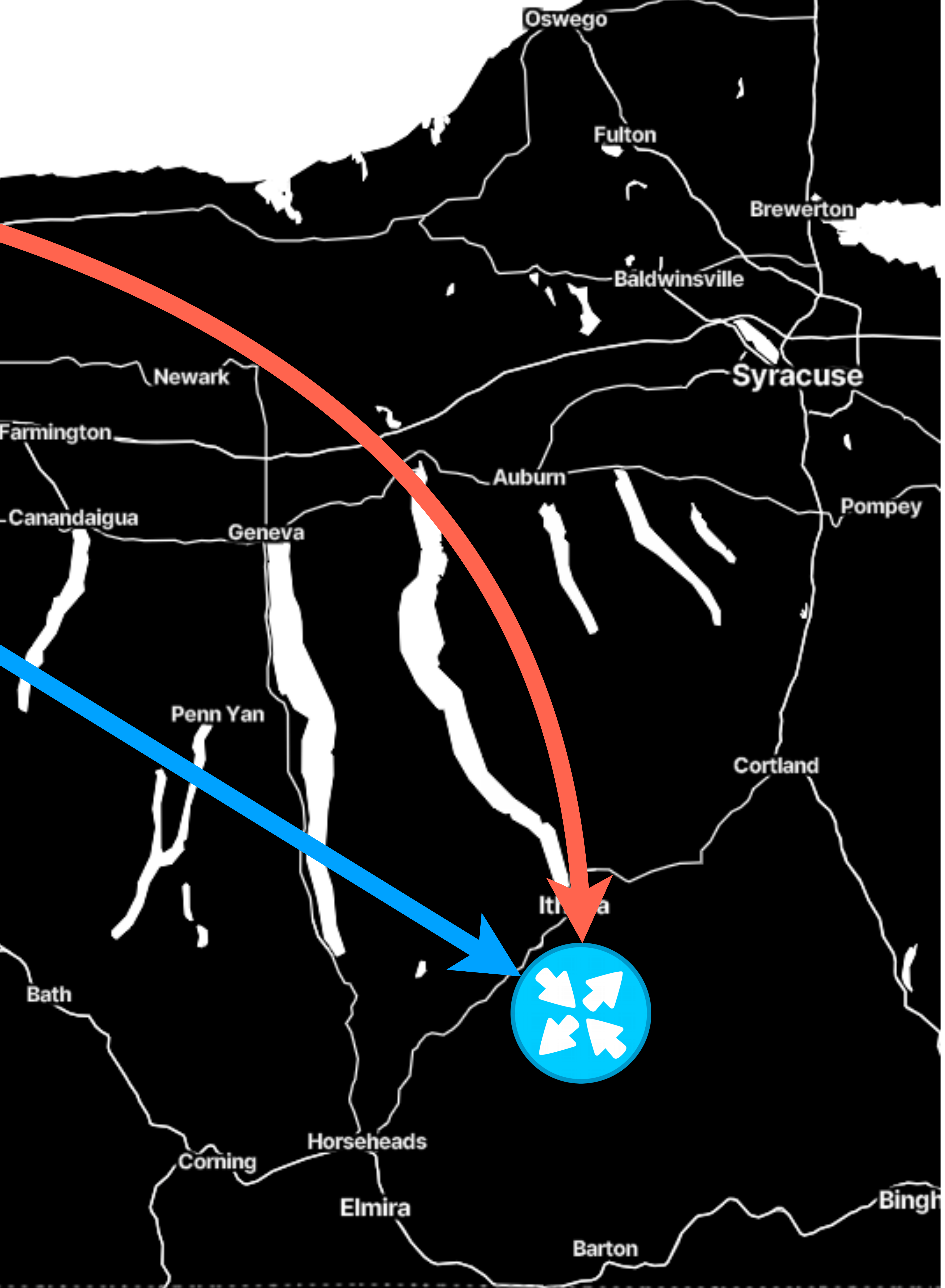

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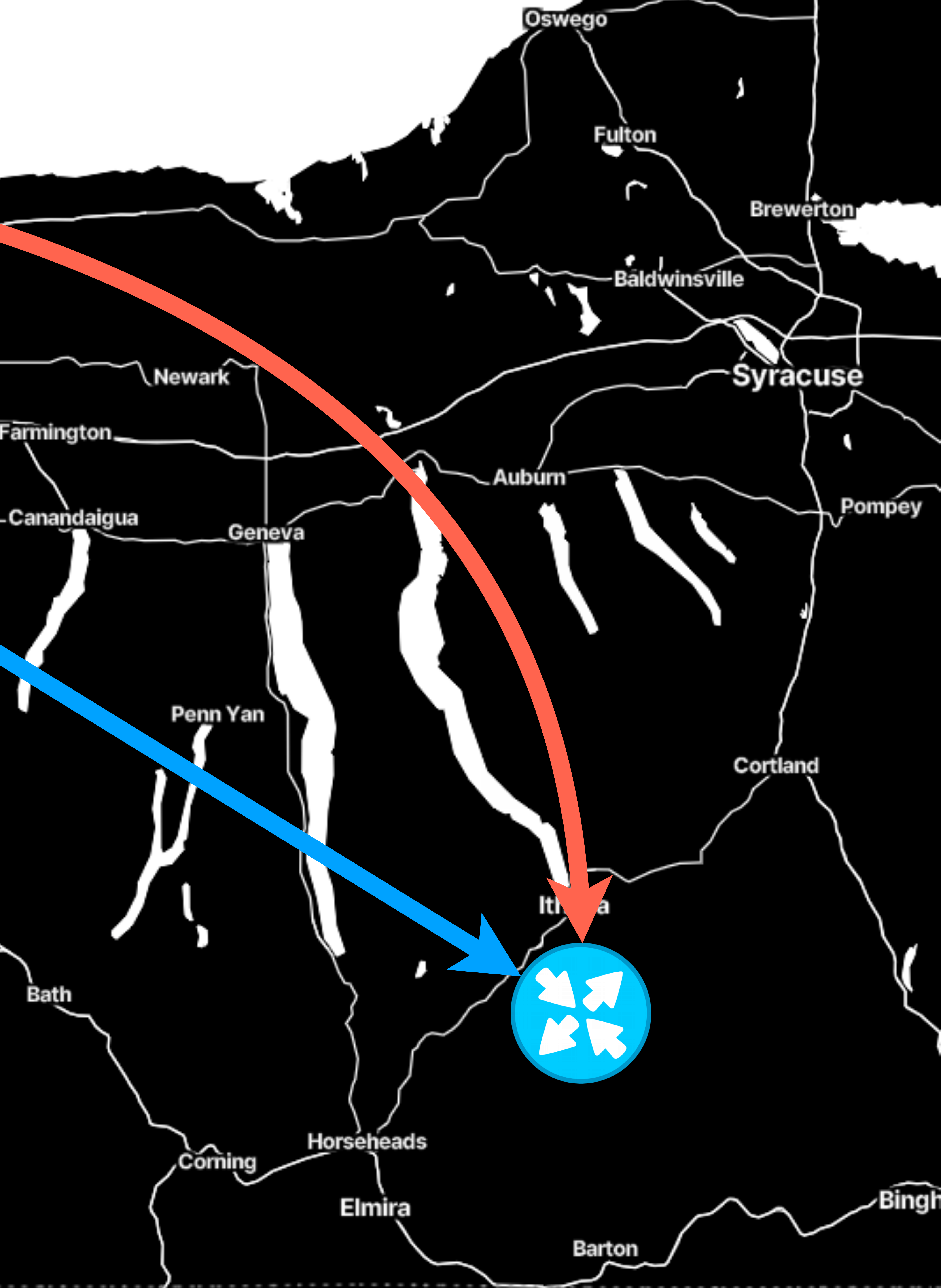

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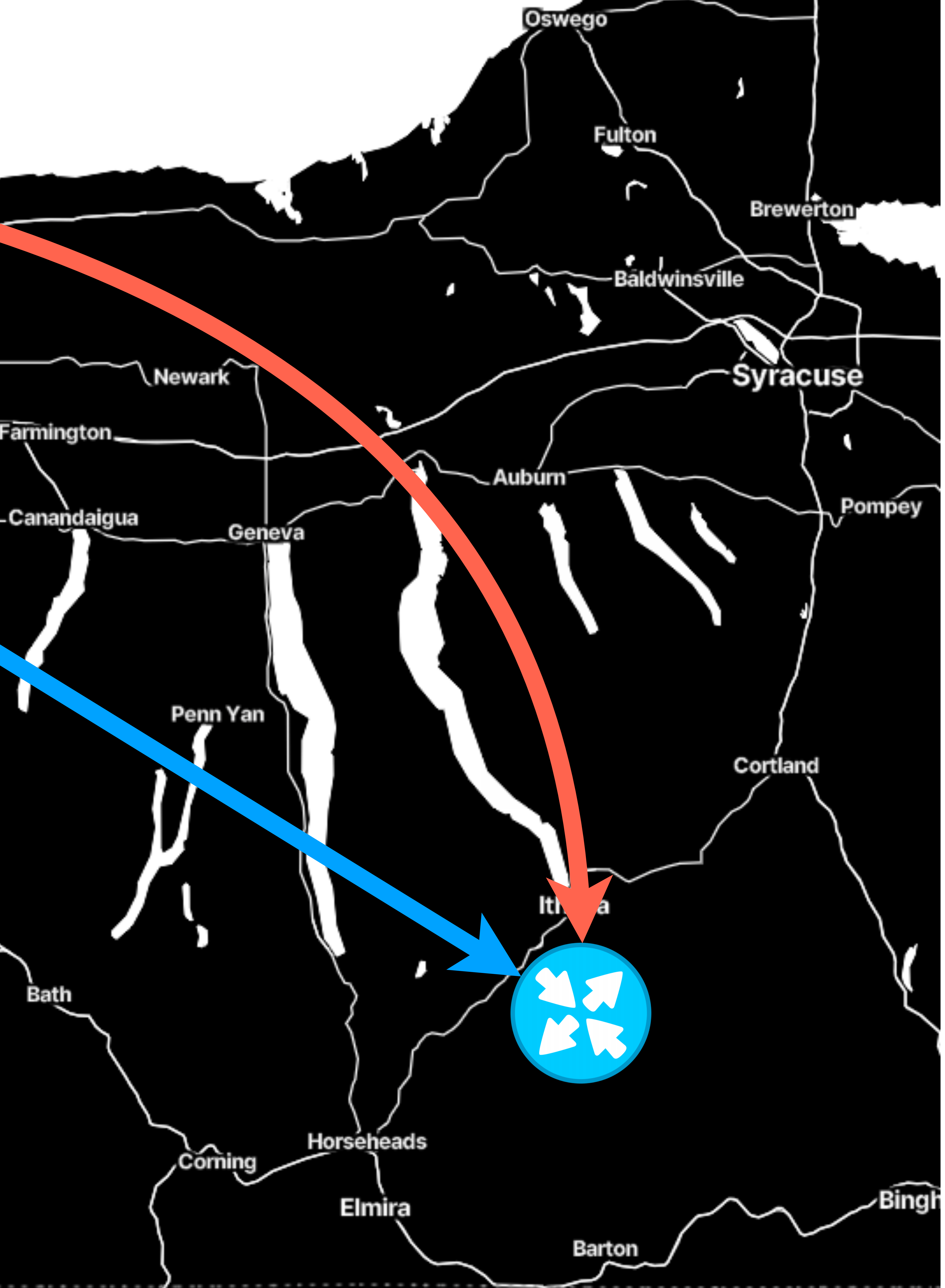
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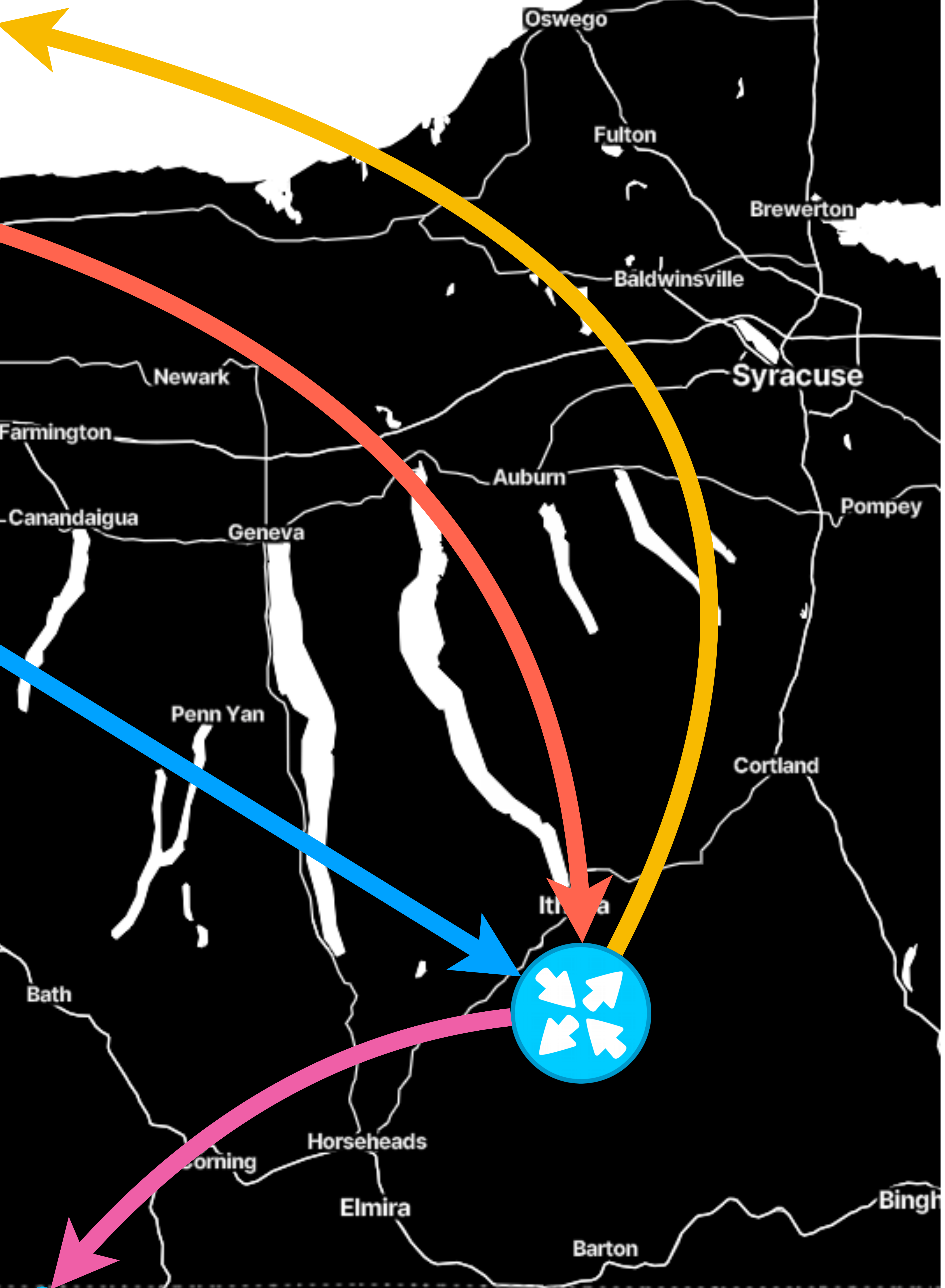
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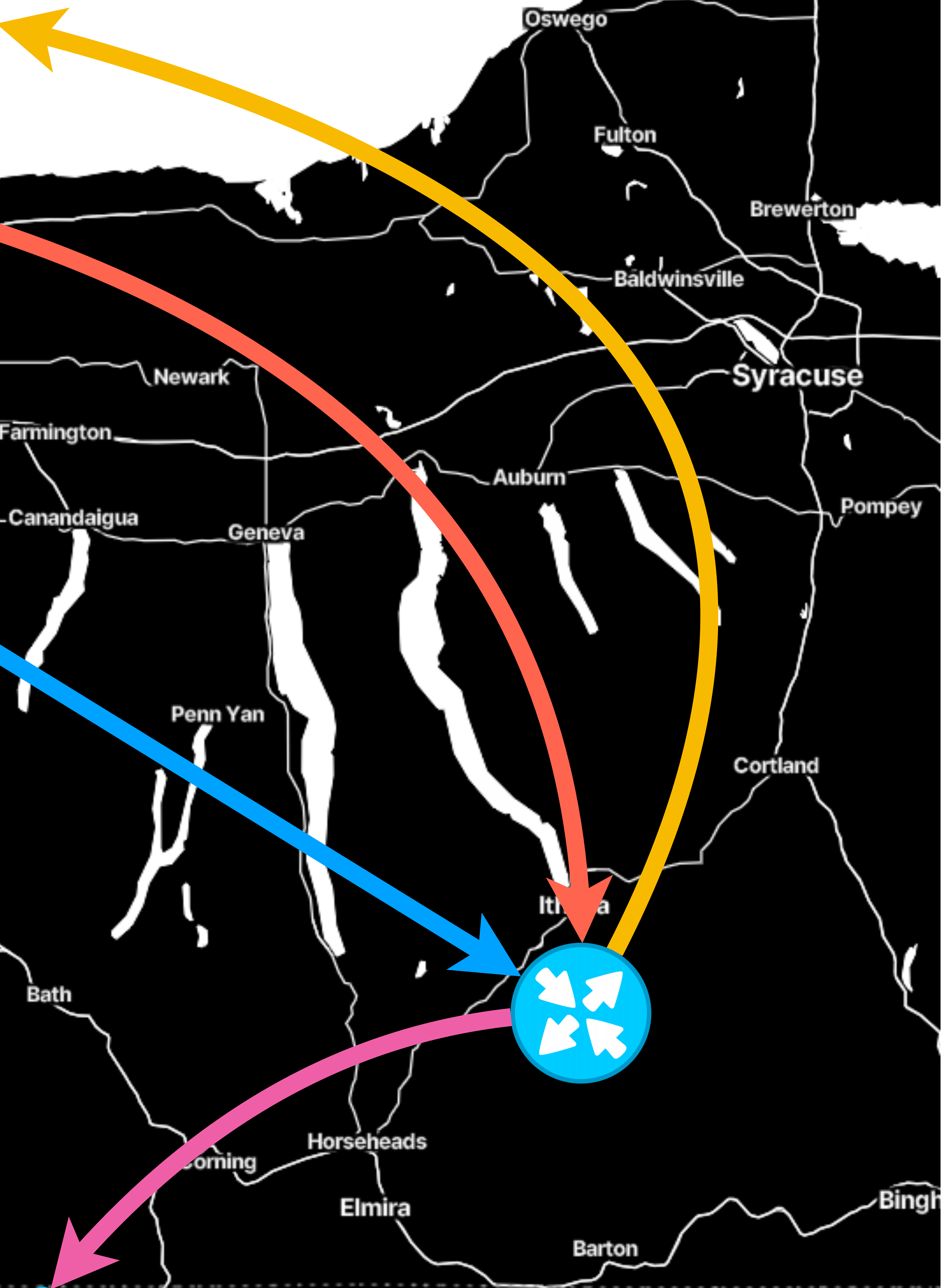

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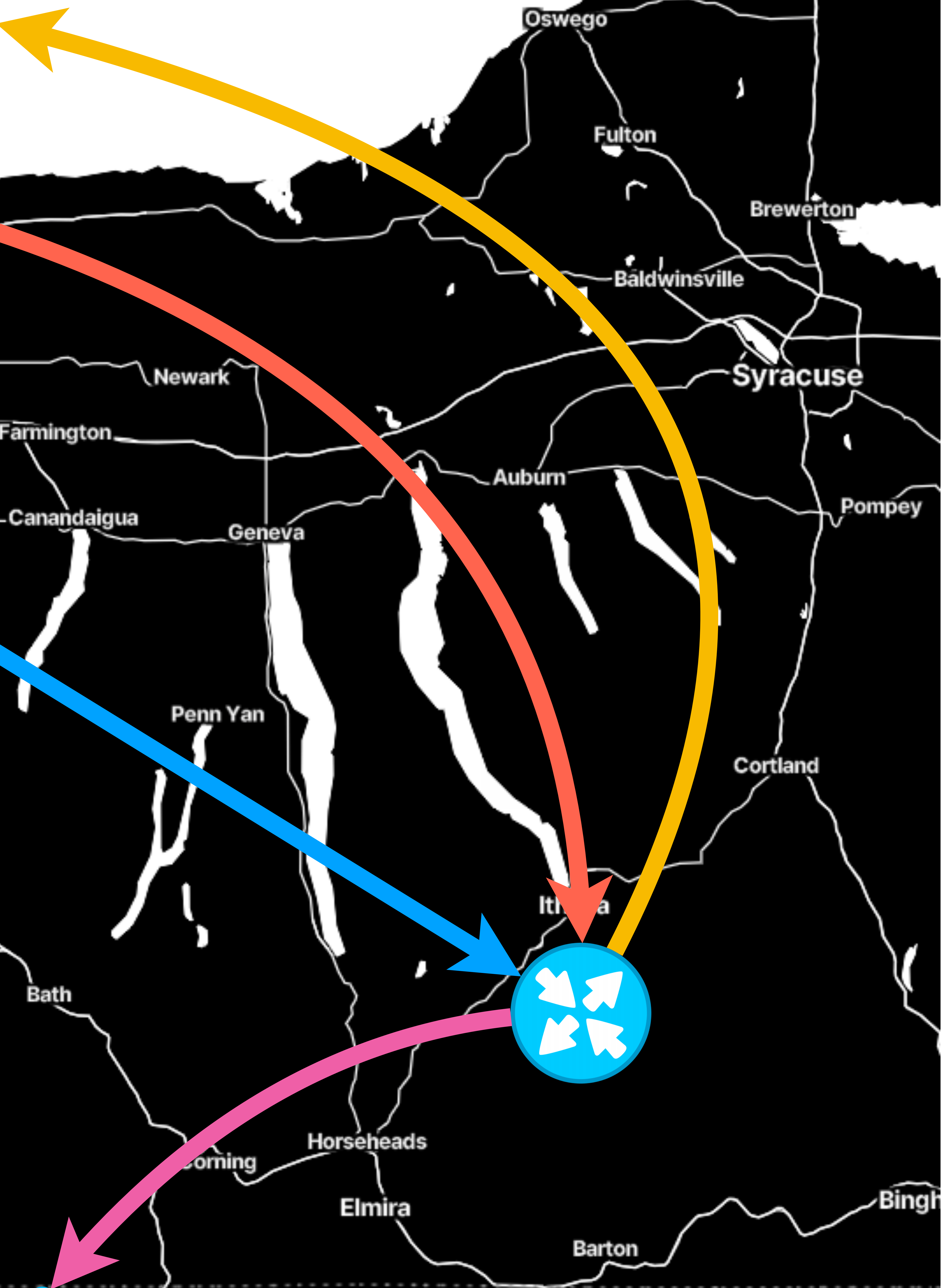

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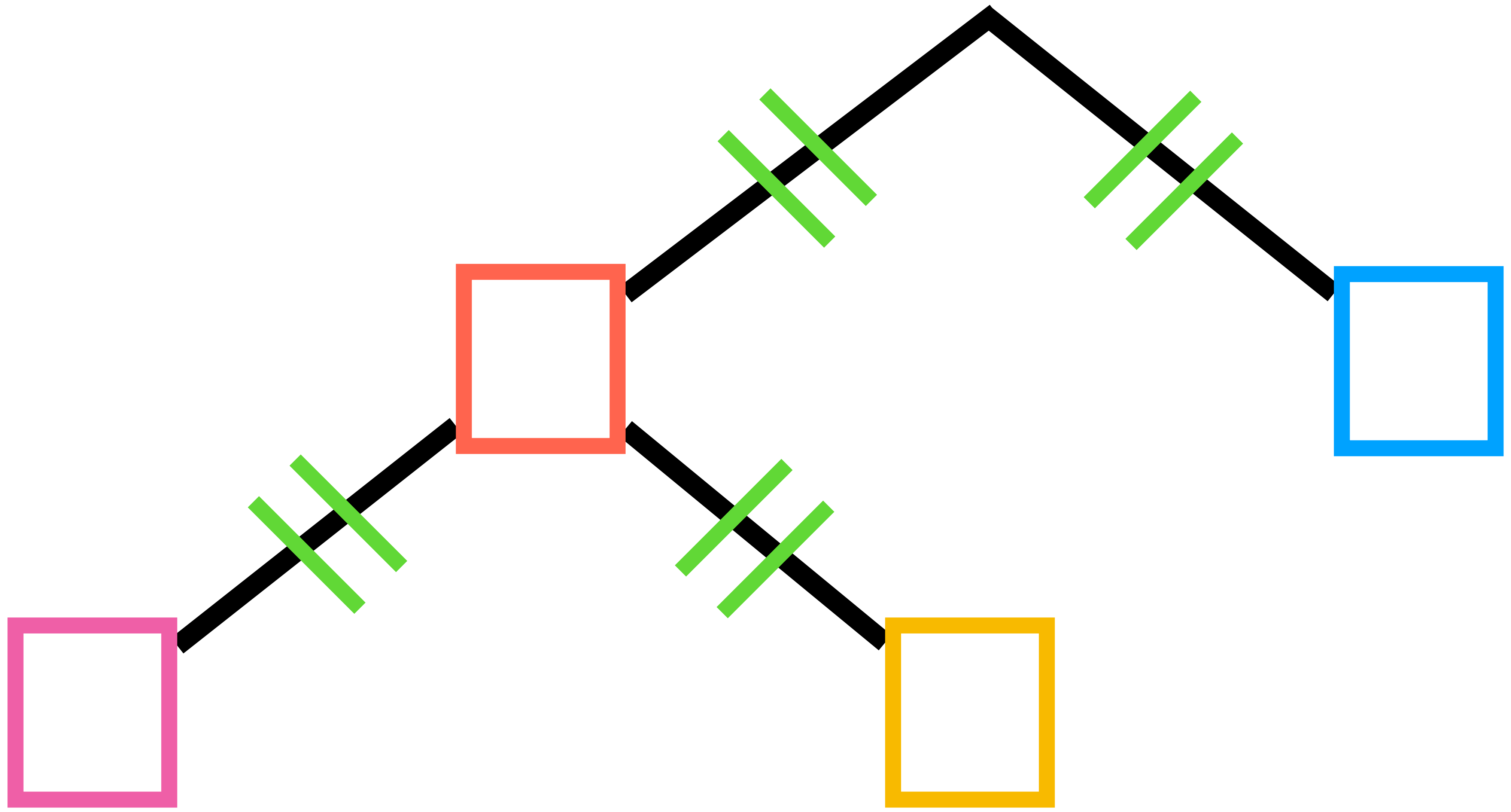

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$B_3, B_2, \blacksquare, \blacksquare, B_1, \blacksquare$

$B_3, B_2, \blacksquare, B_1, \blacksquare, \blacksquare$

Goal:
interleave R and B;
interleave P and T.

$B_3, B_2, P_2, T_1, B_1, P_1$
$B_3, B_2, P_2, B_1, T_1, P_1$

$T_1 \longrightarrow B_3, B_2, P_2, B_1, P_1$

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B_3, P_2, B_2, T_1, B_1, P_1

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B_3, , B_2, , B_1,

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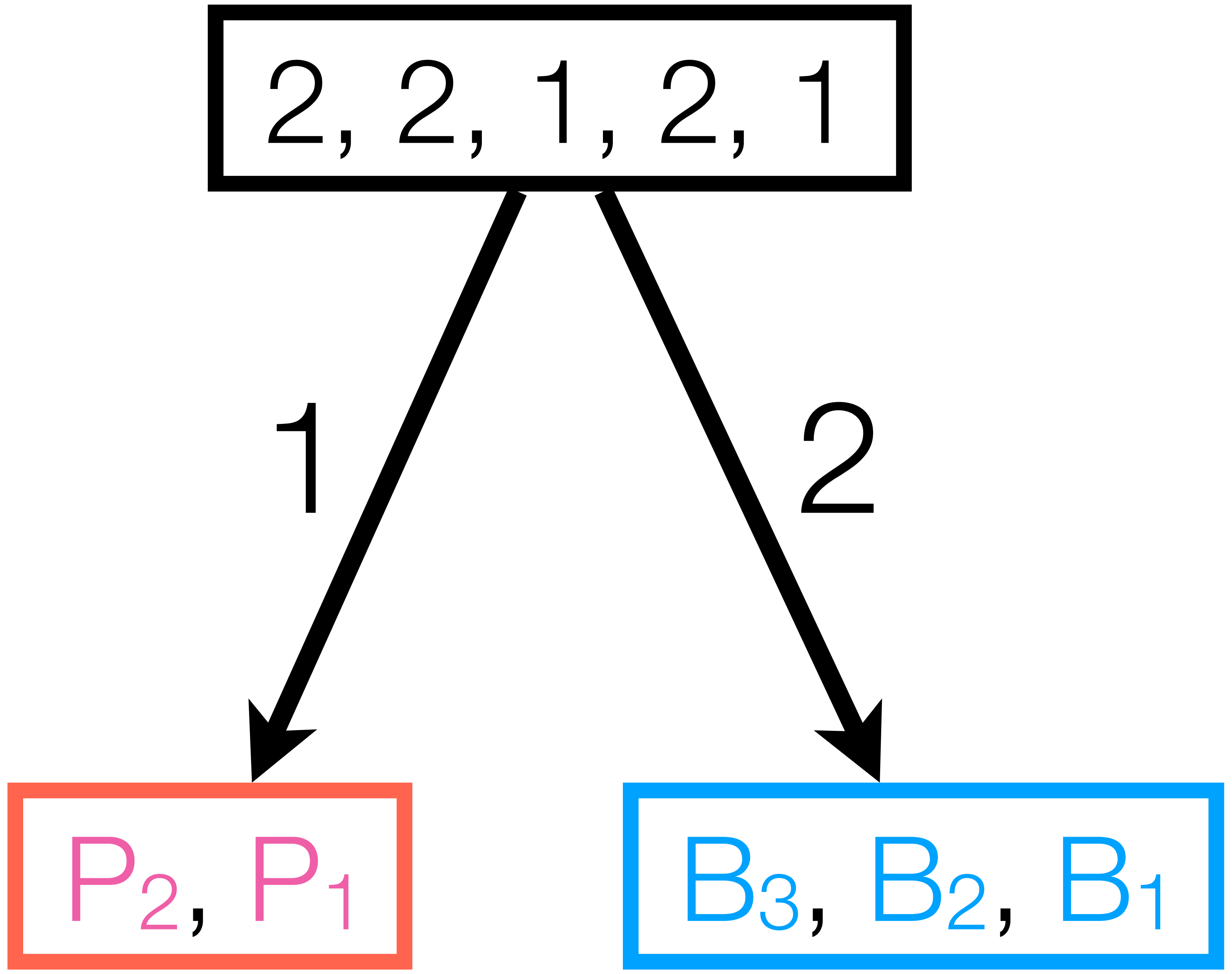
Enqueueing a packet can require the reordering of buffered packets.

No PIFO can do this.

Introducing: PIFO trees

Interleave R and B;
interleave P and T.



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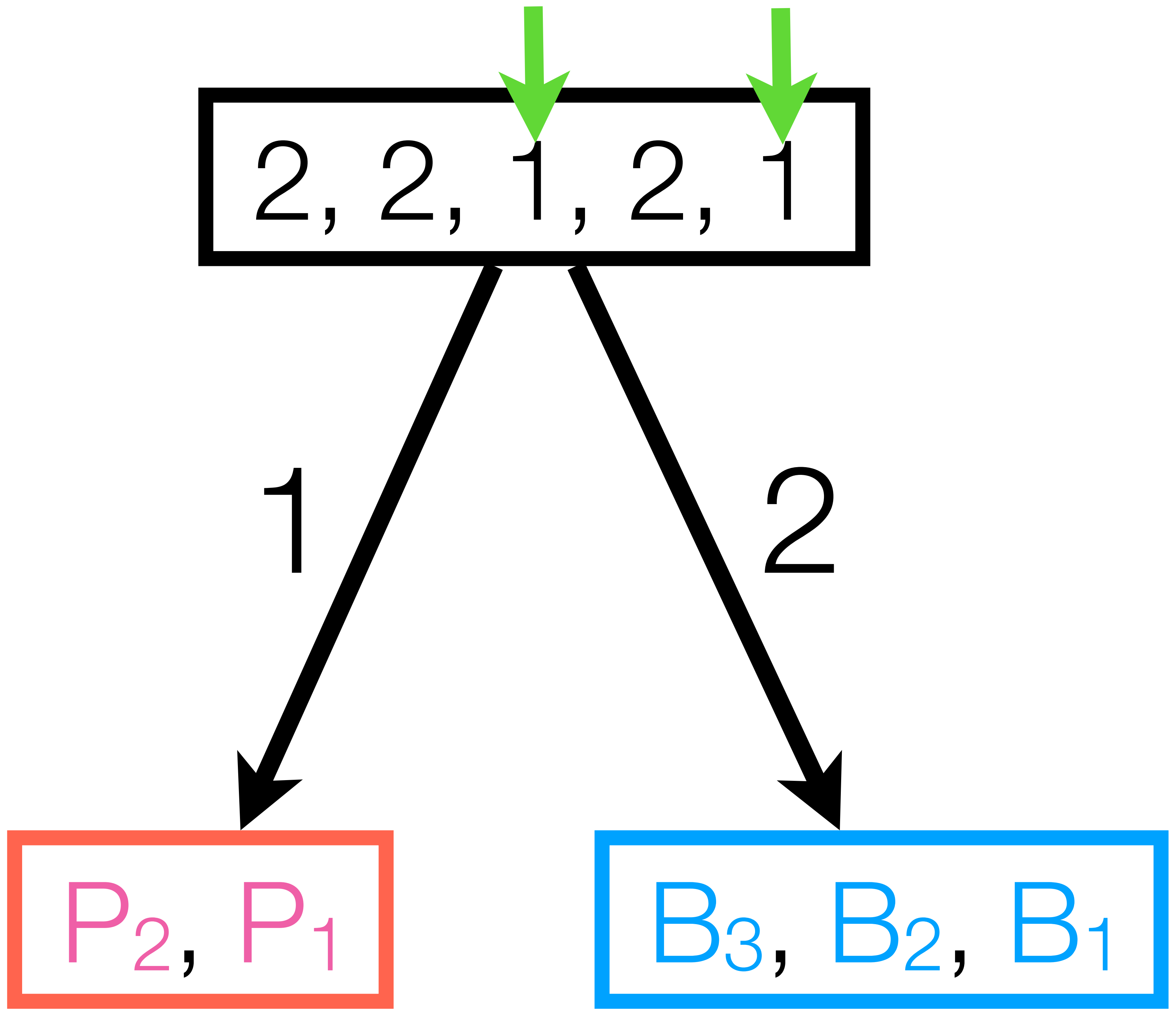

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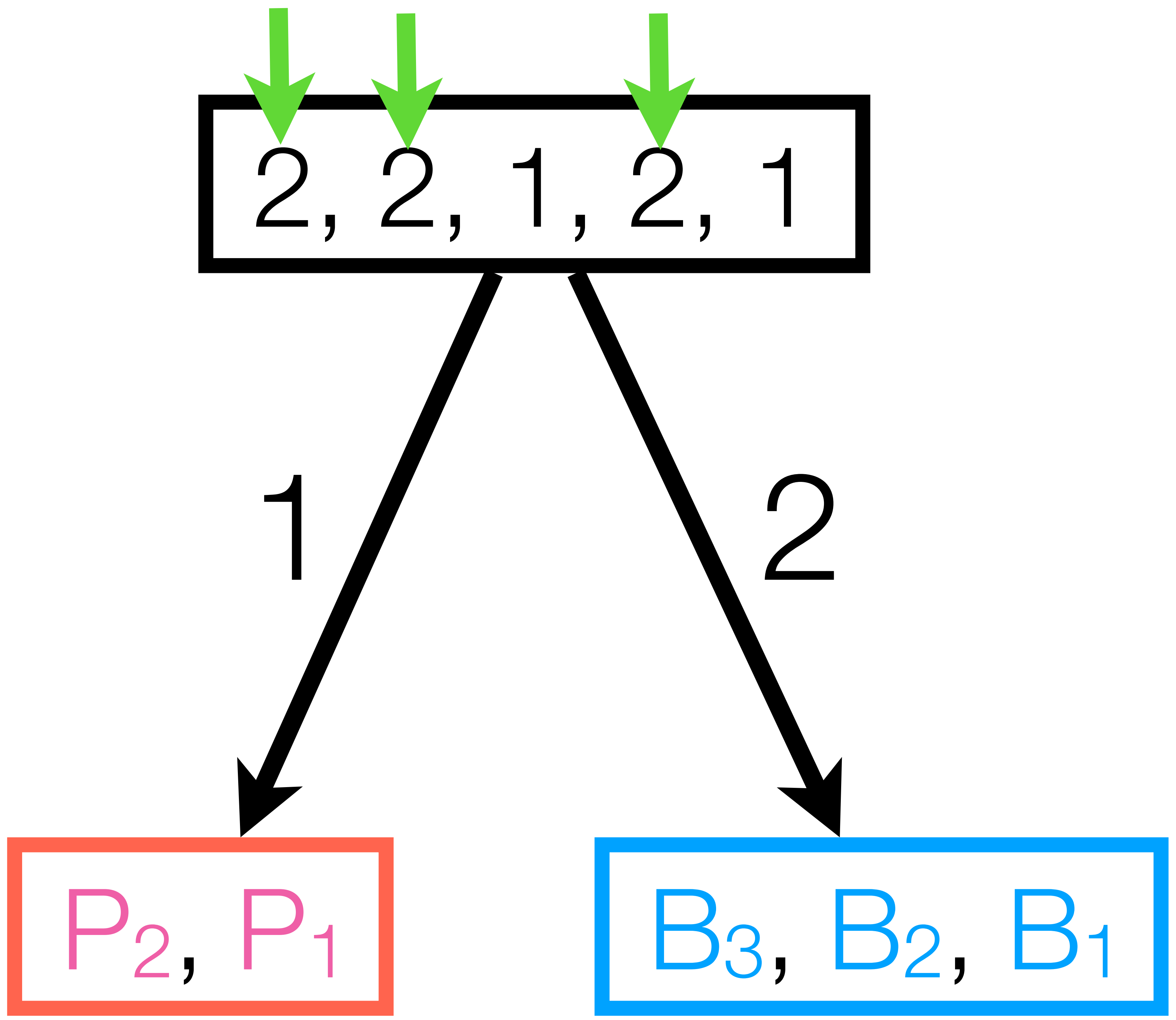

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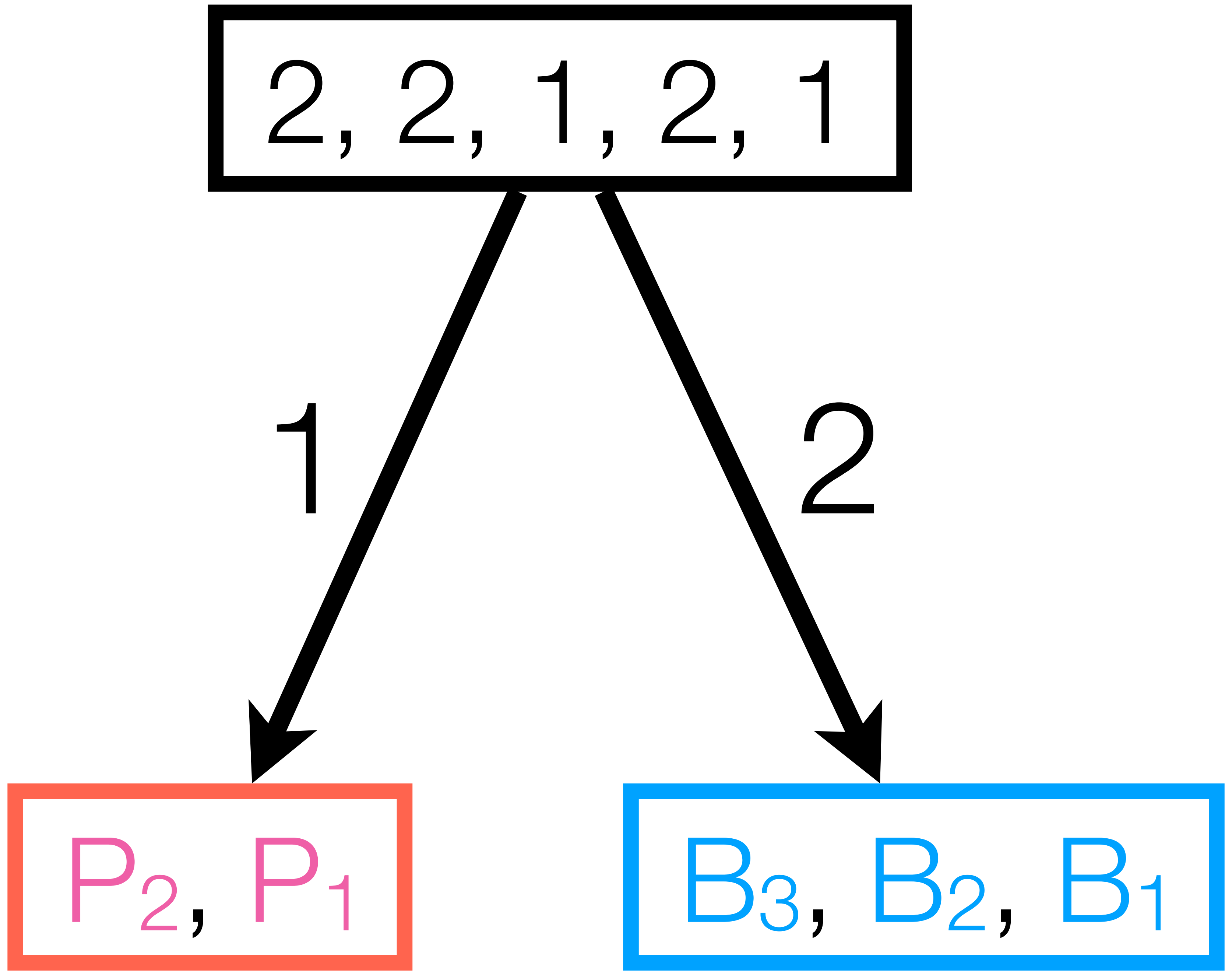

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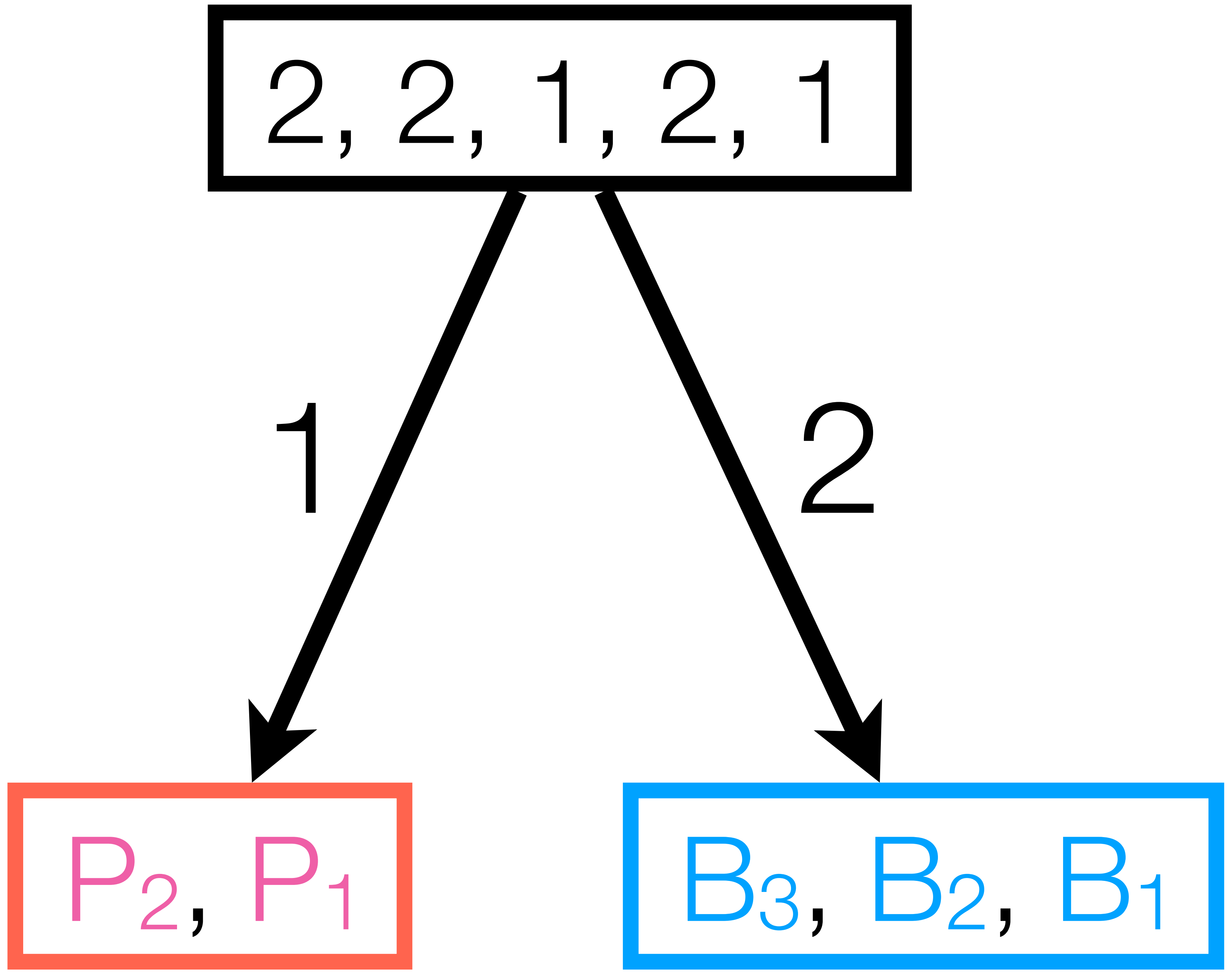

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How do we pop it?

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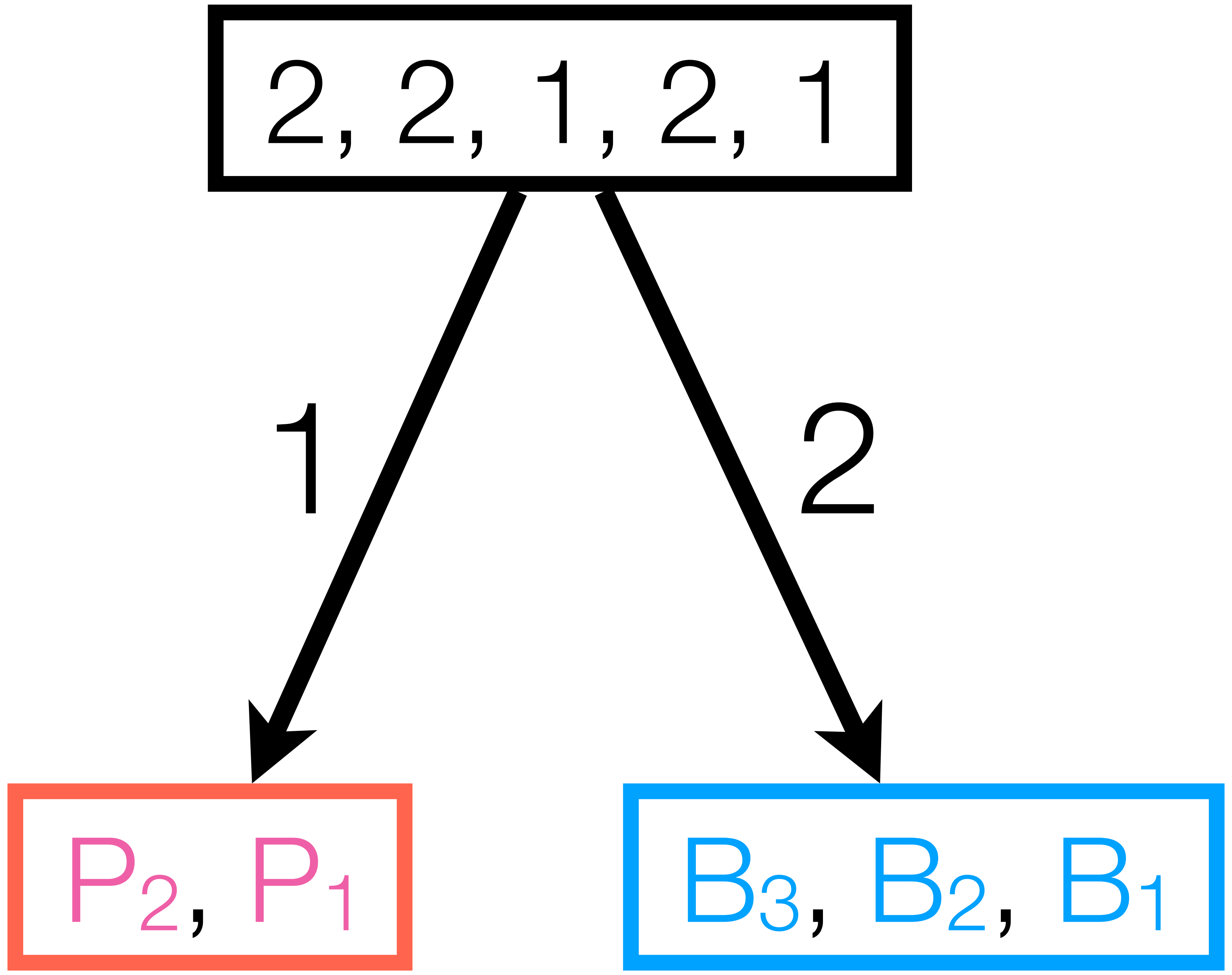

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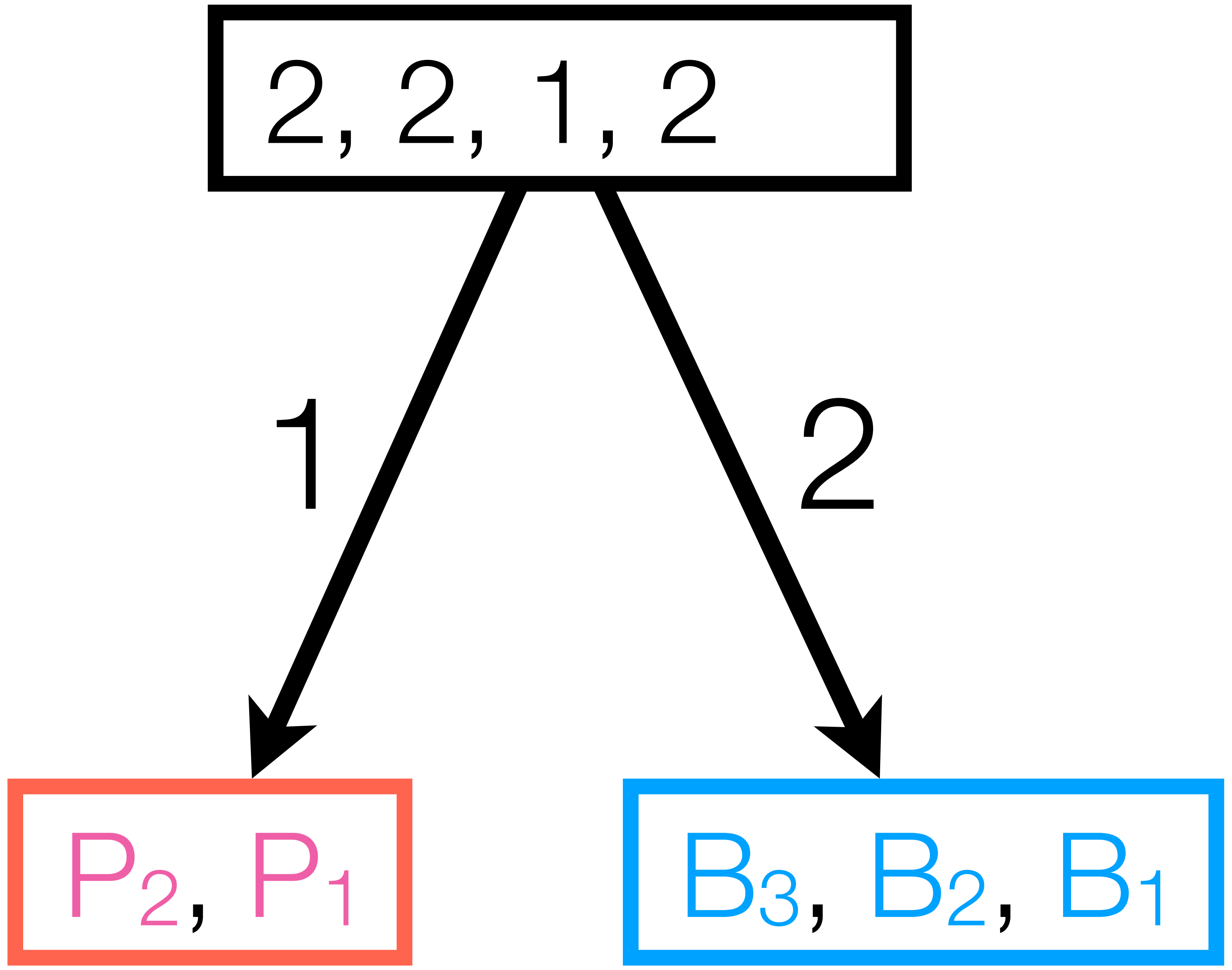

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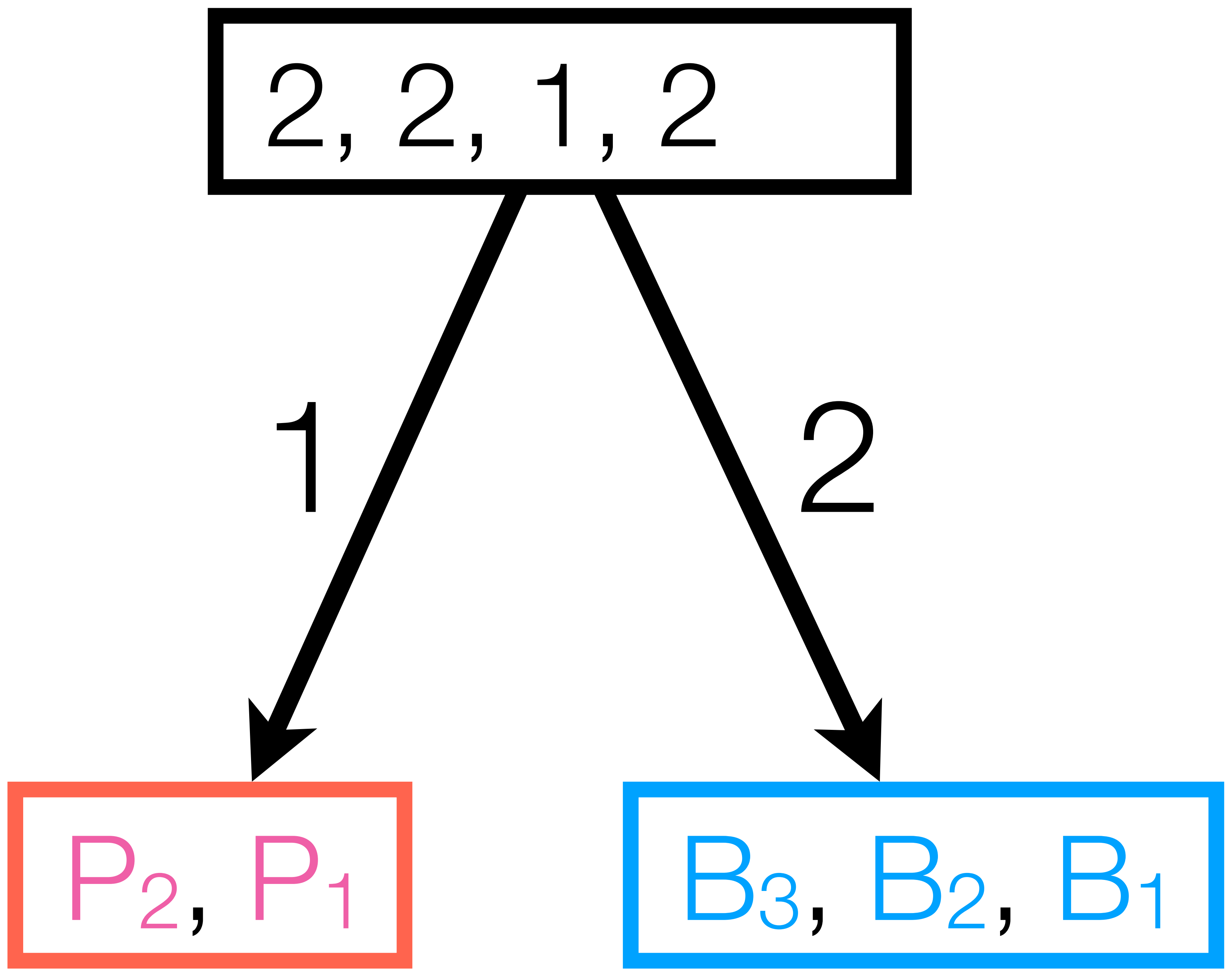

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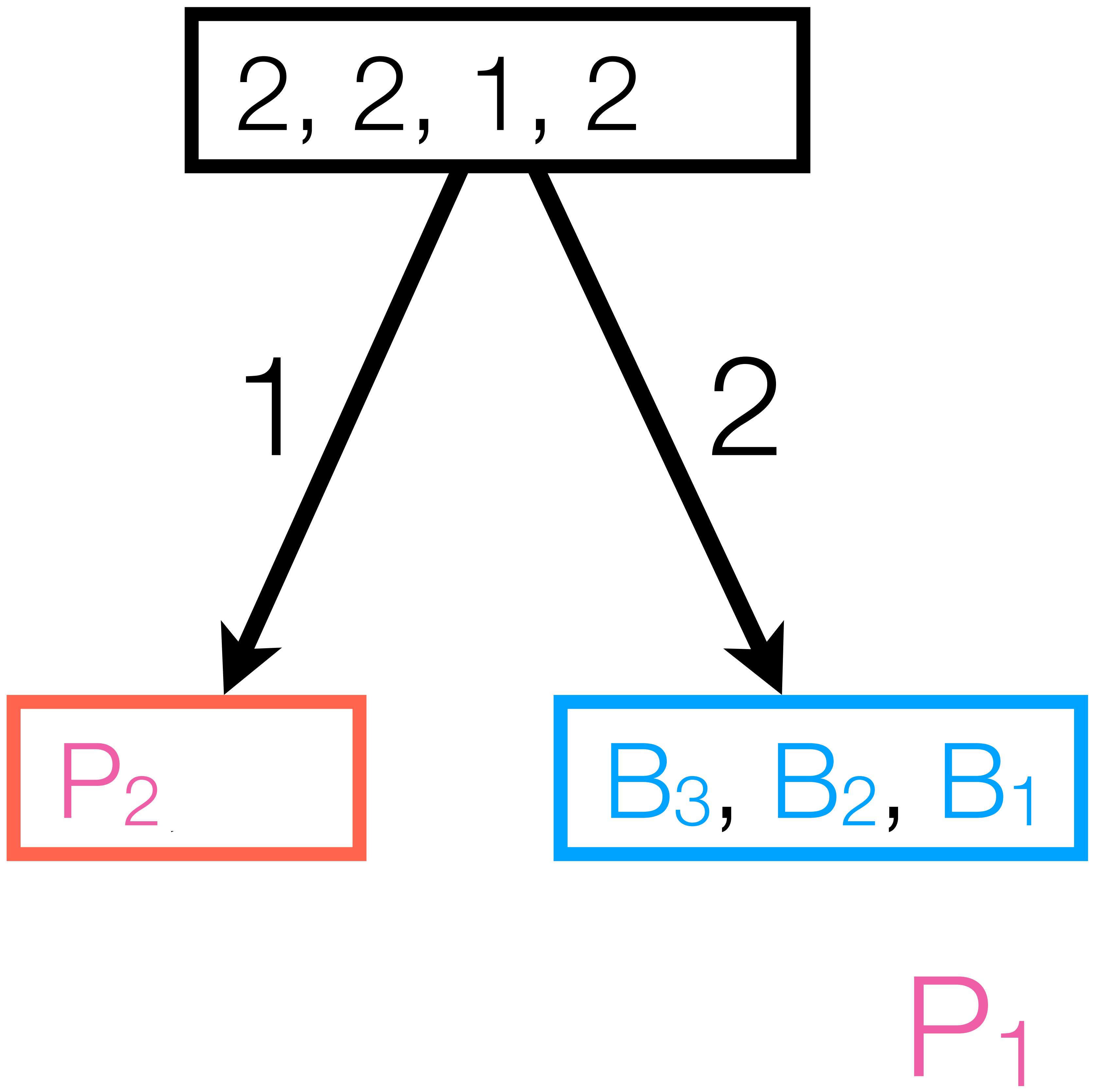

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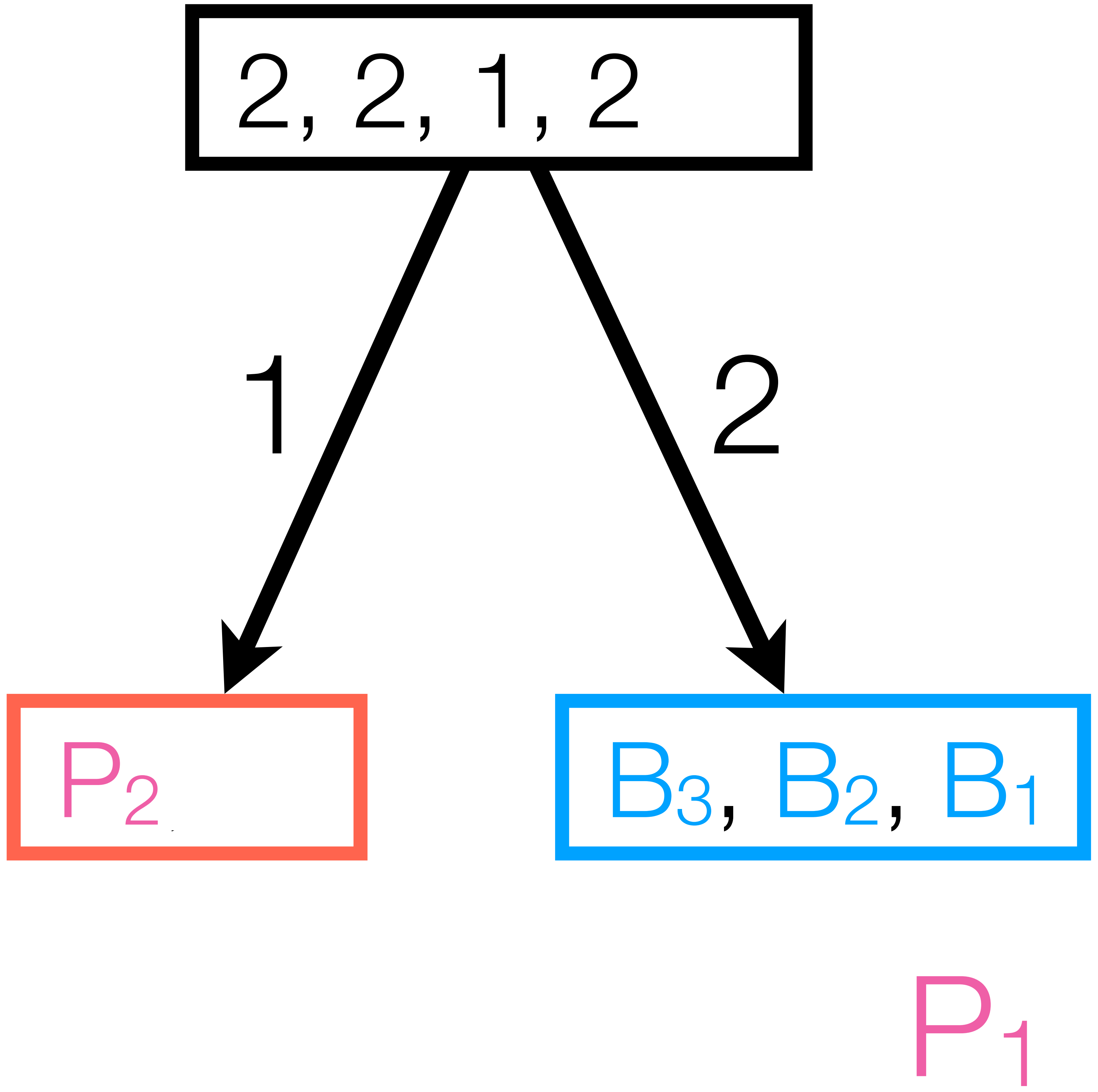

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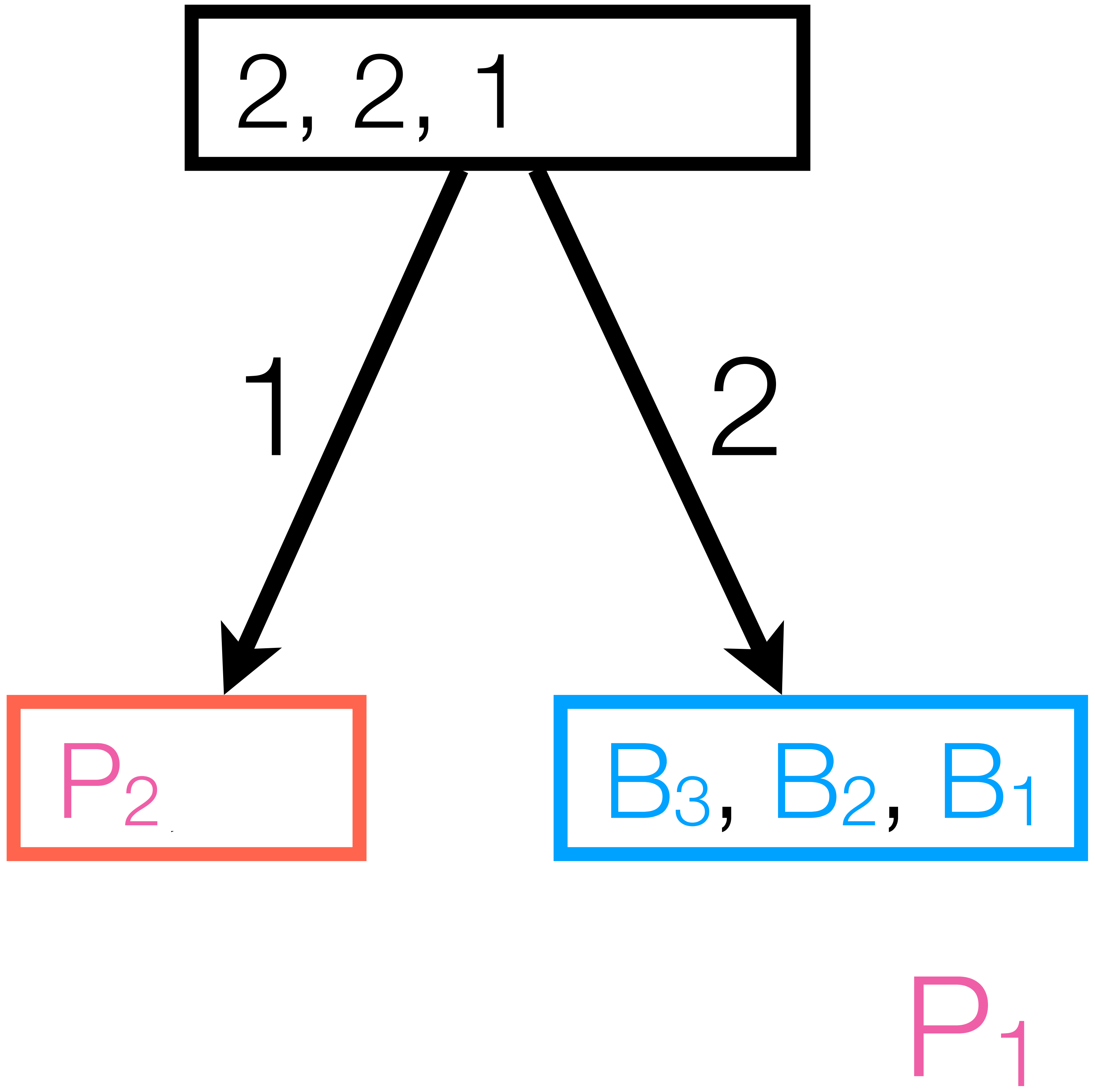

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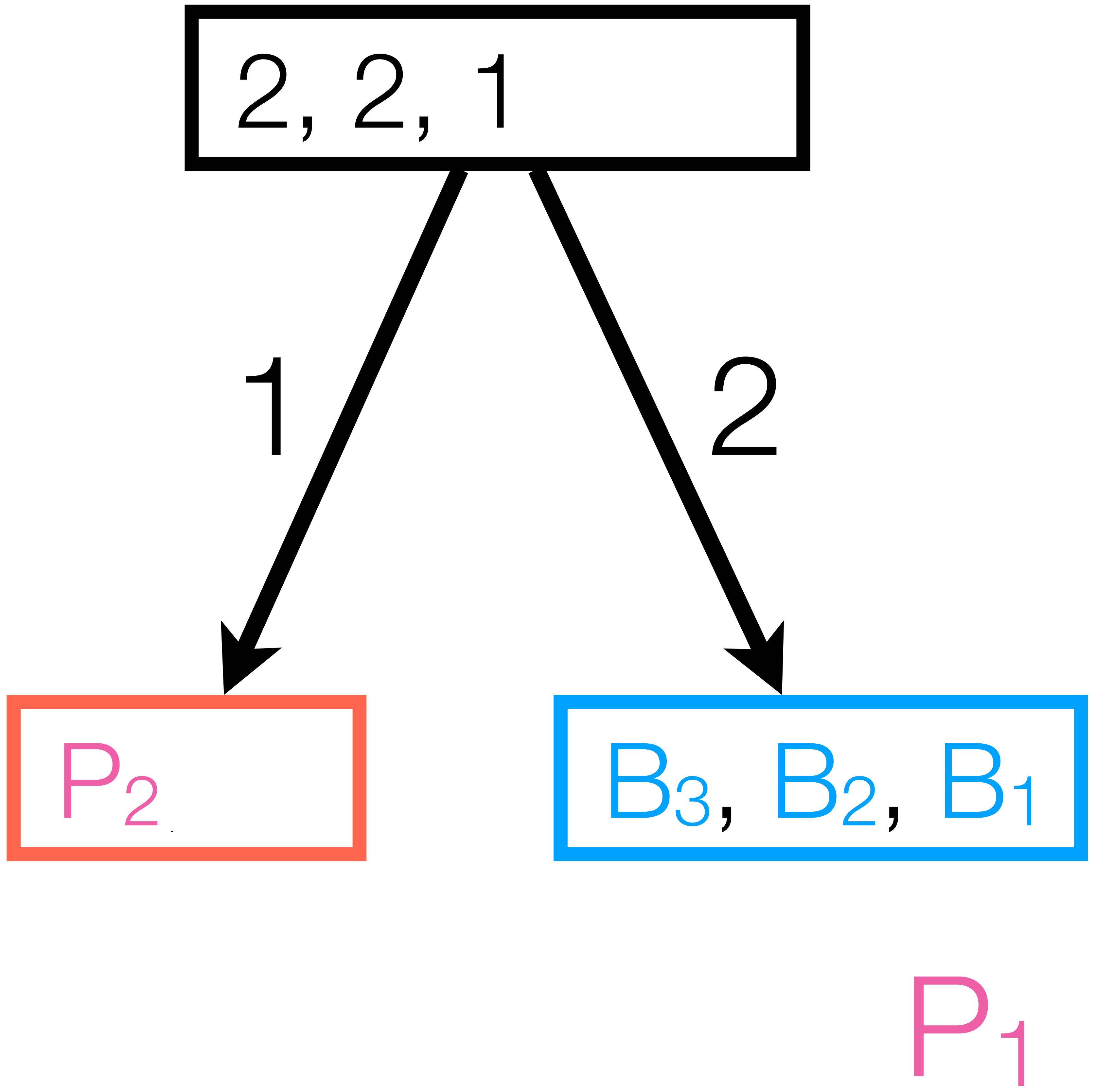

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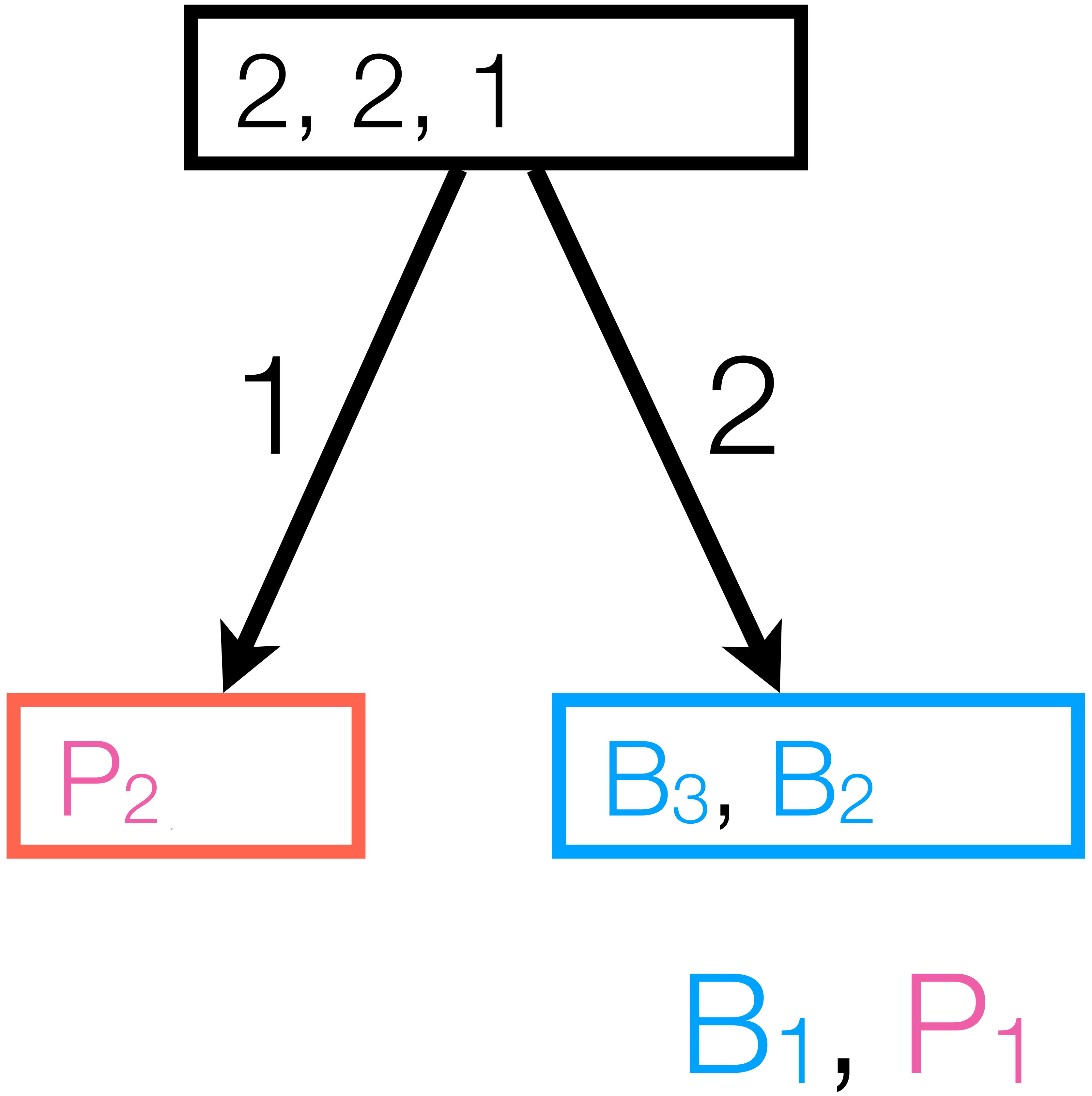

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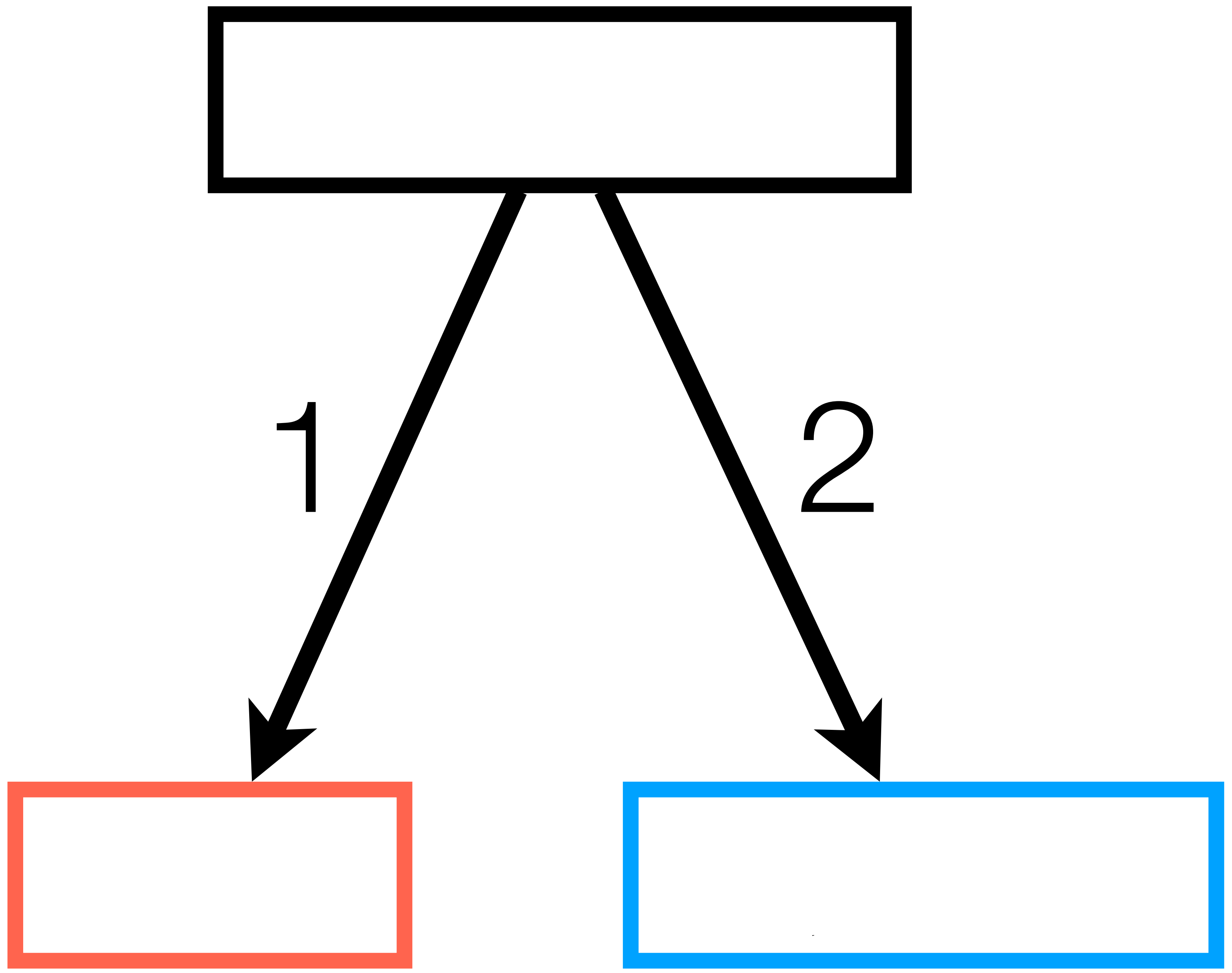

B_3, B_2, P_2, B_1, P_1

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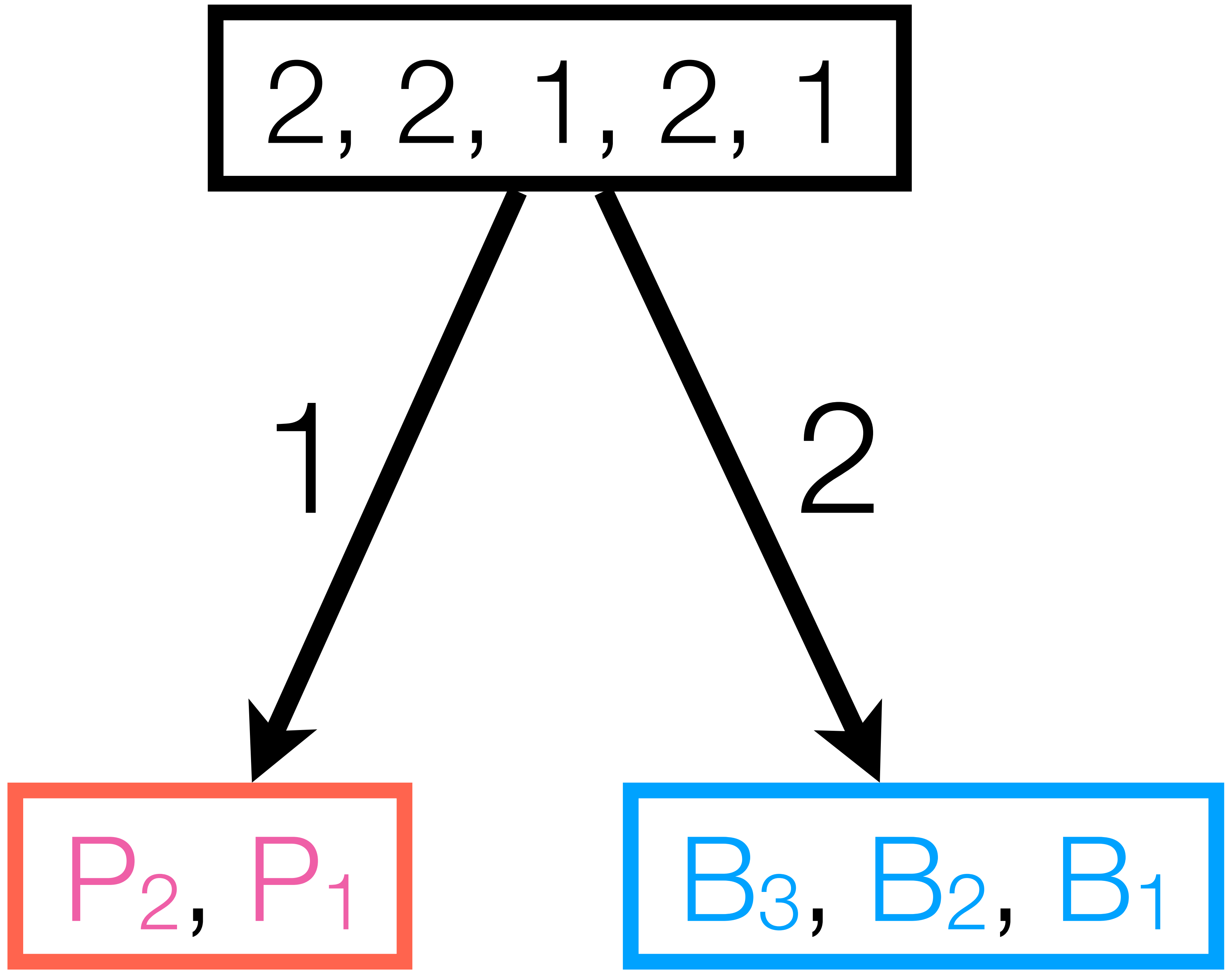

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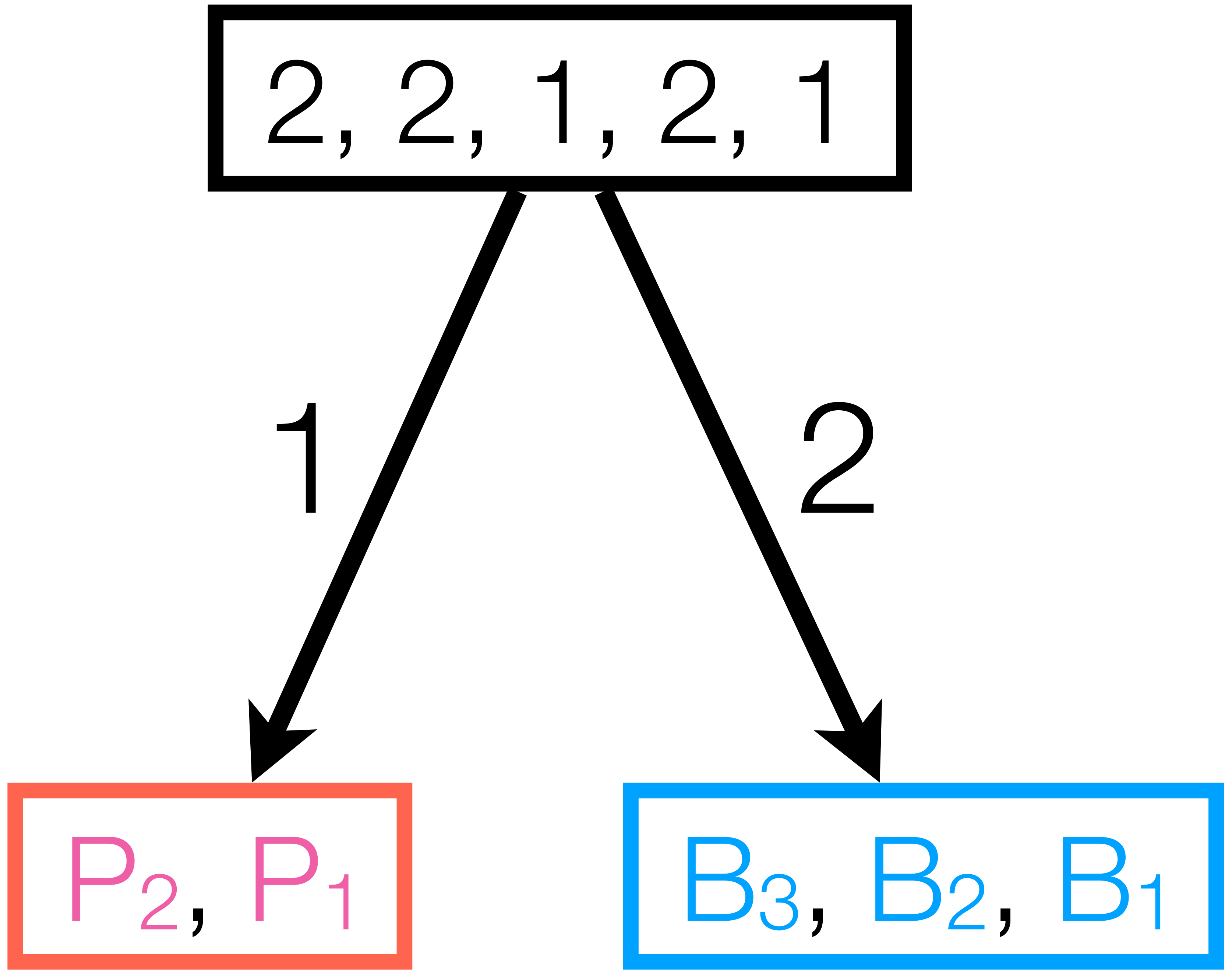

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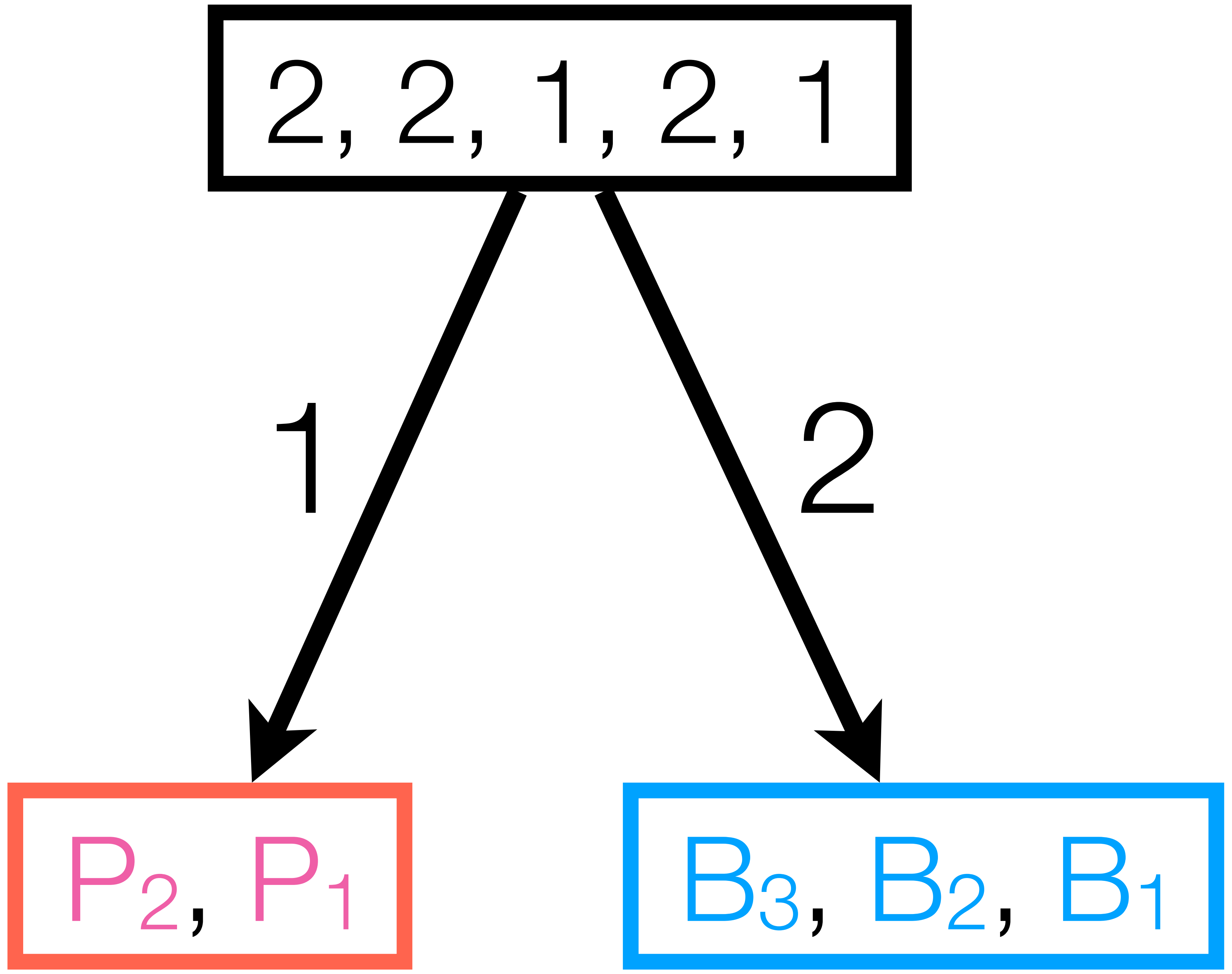

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This behaves like a queue!

How do we pop it?

How do we push into it?

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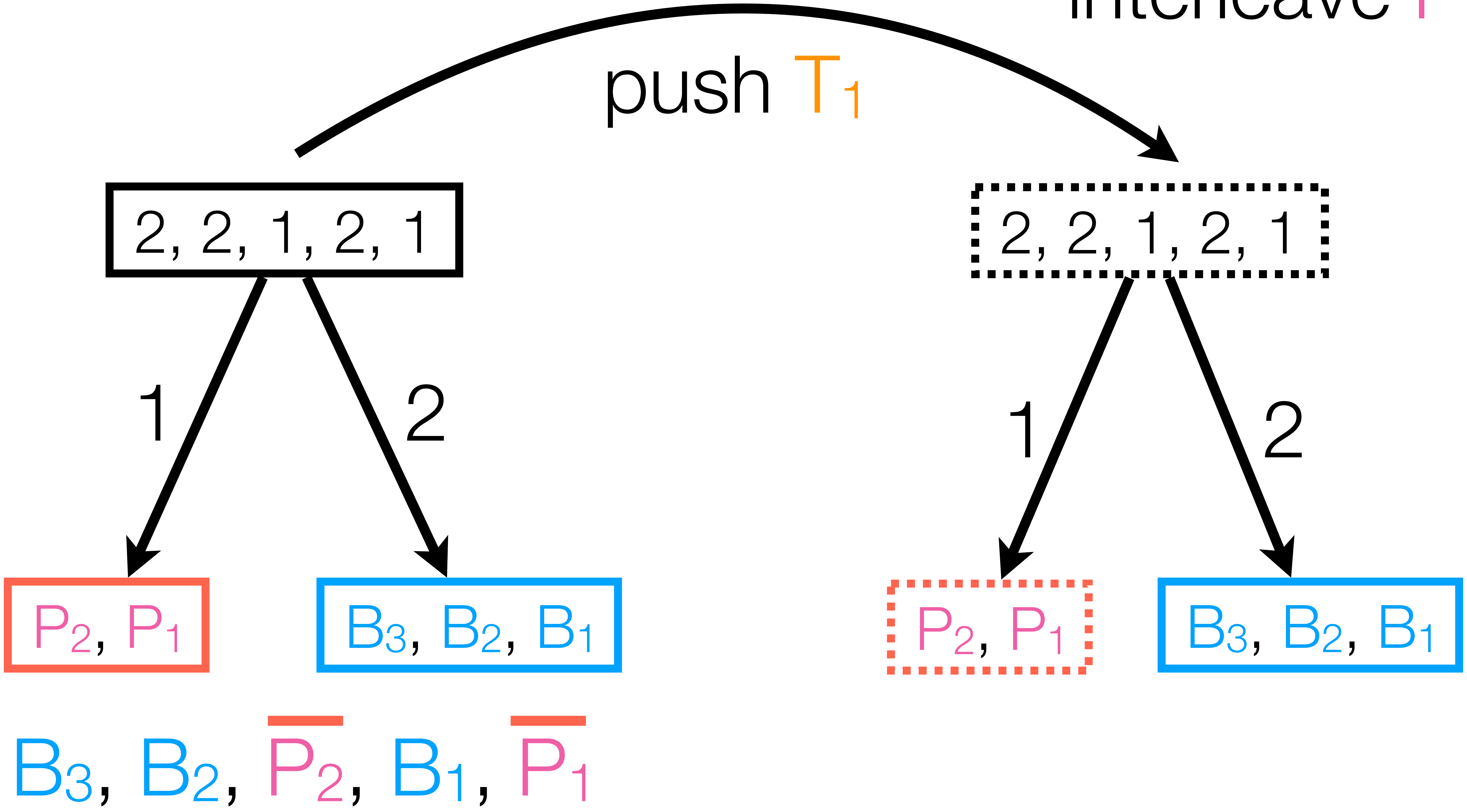

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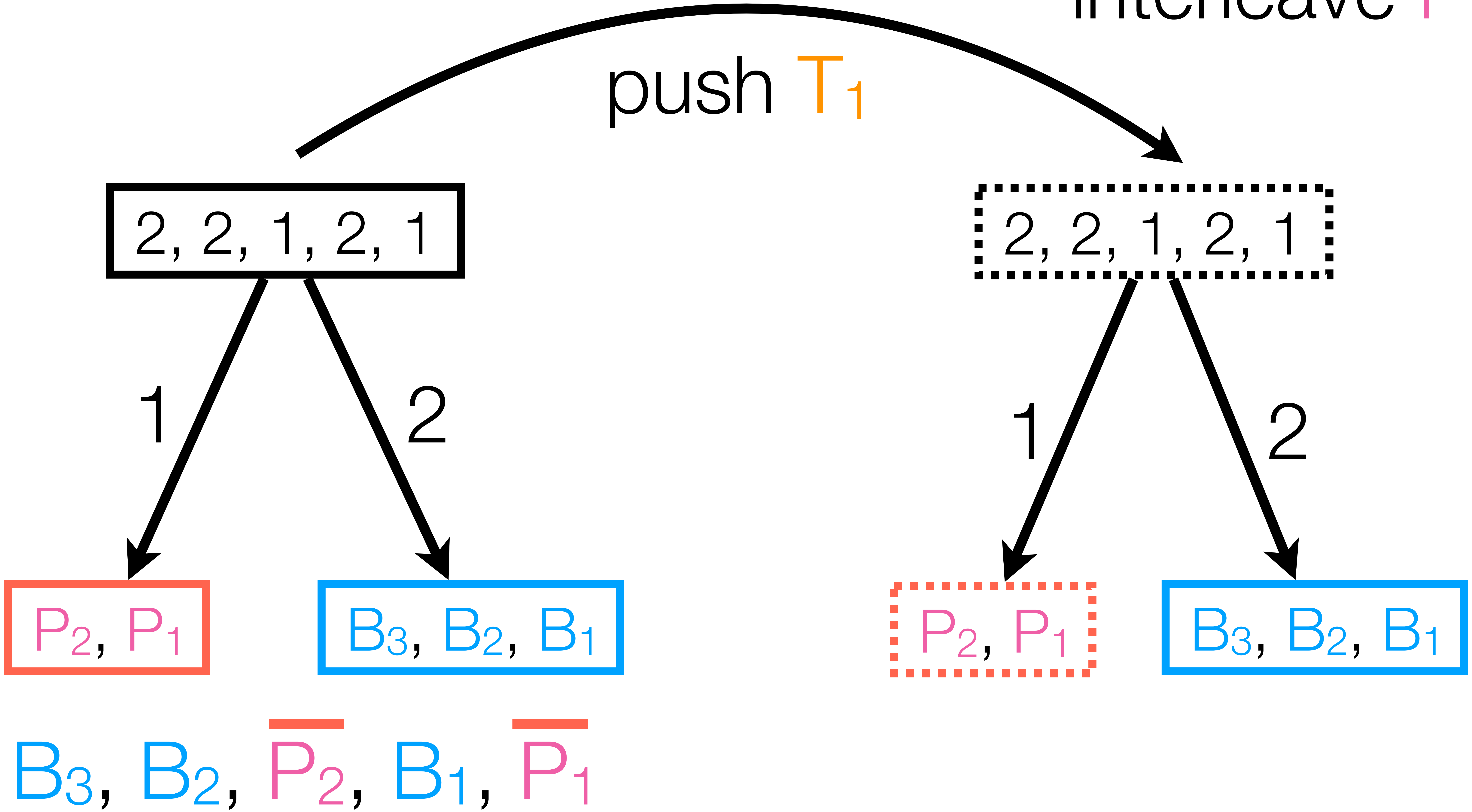

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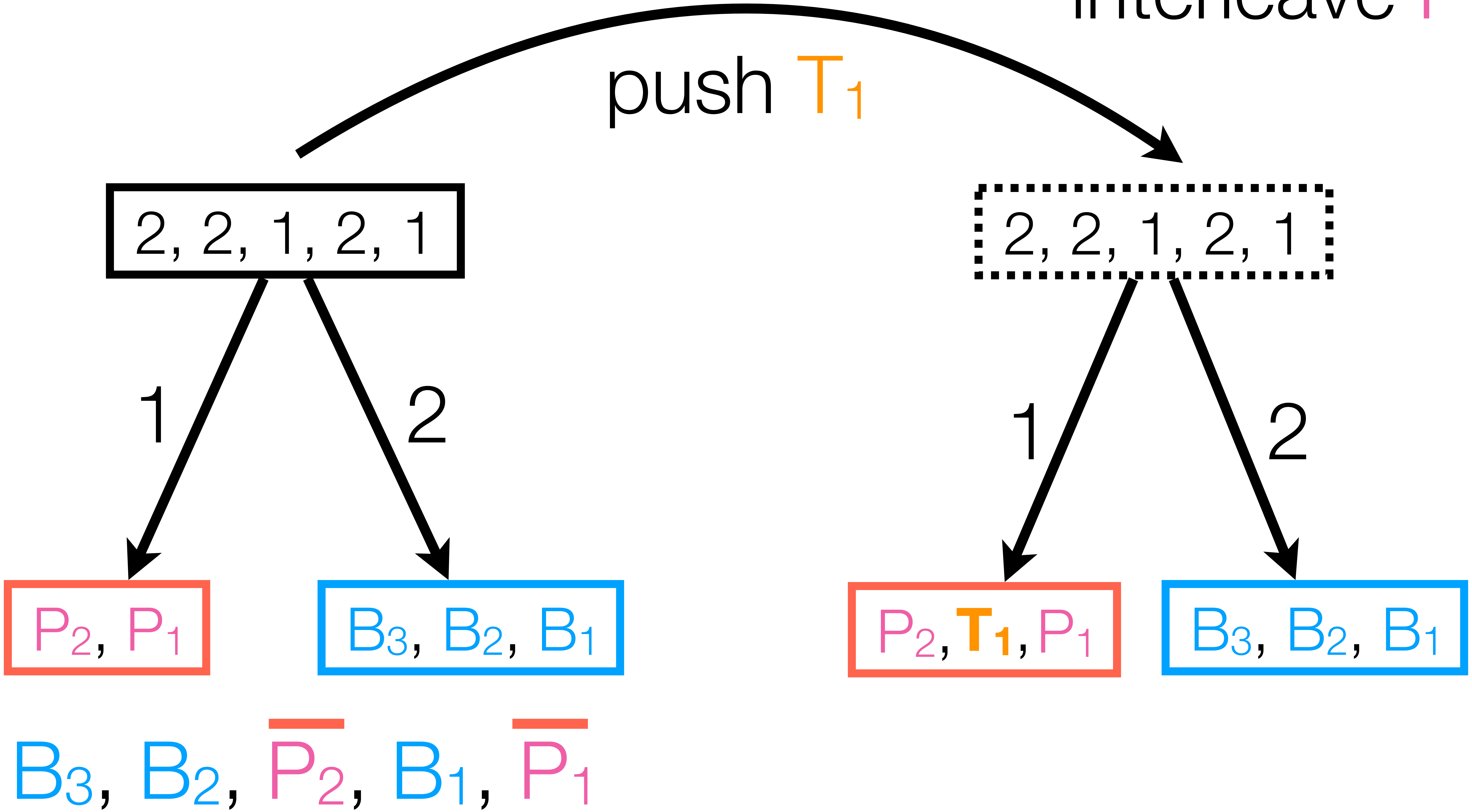

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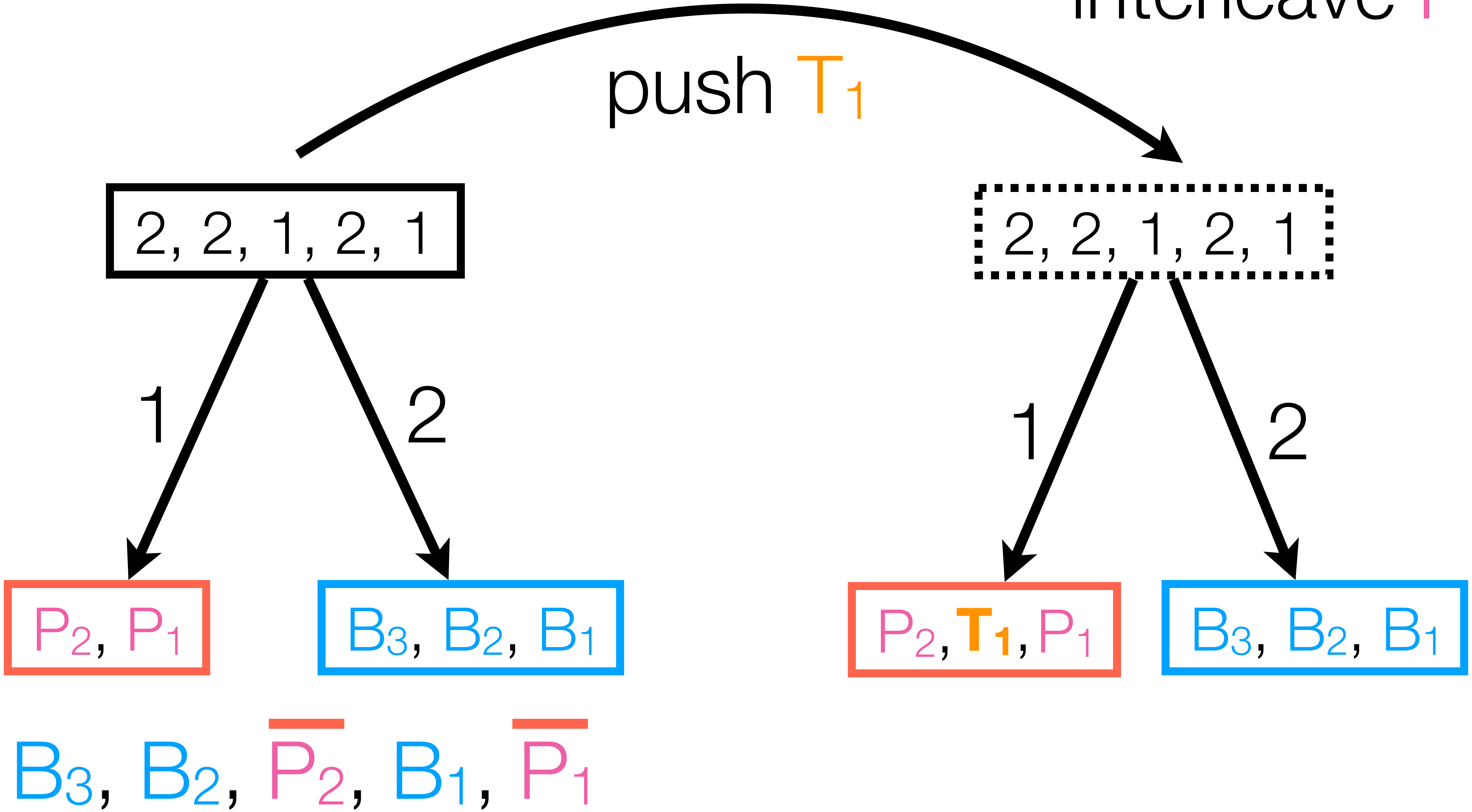

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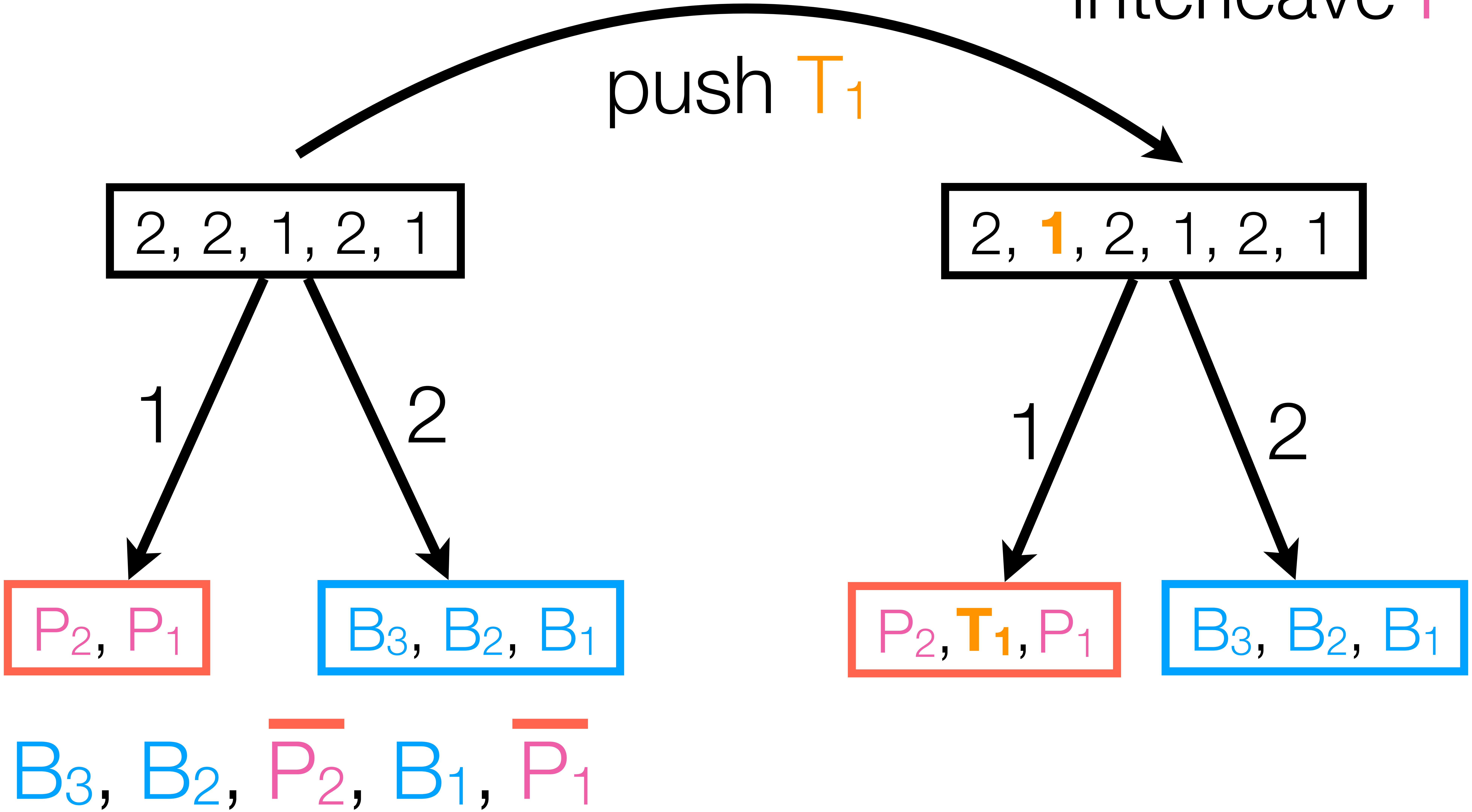

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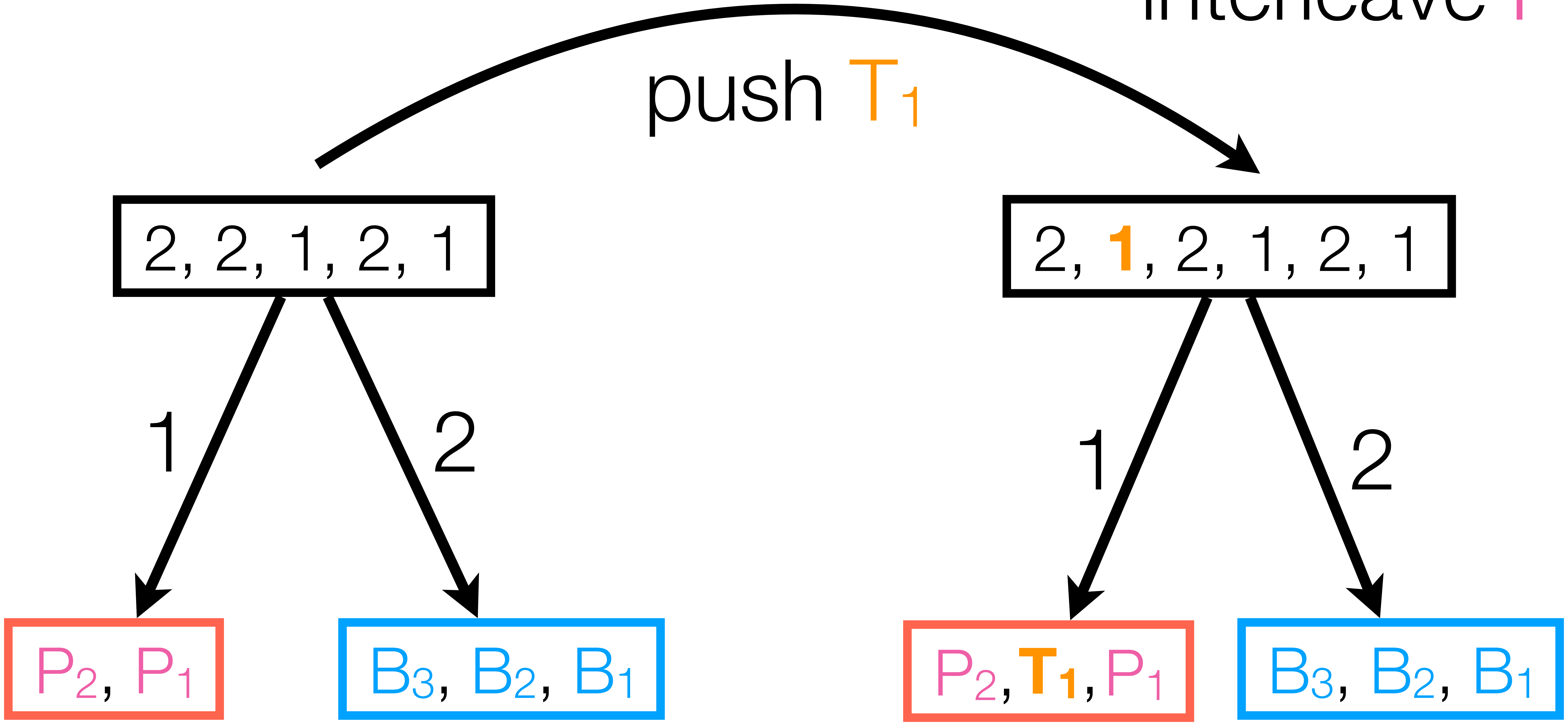

B_3, B_2, $\overline{P_2}$, B_1, $\overline{P_1}$

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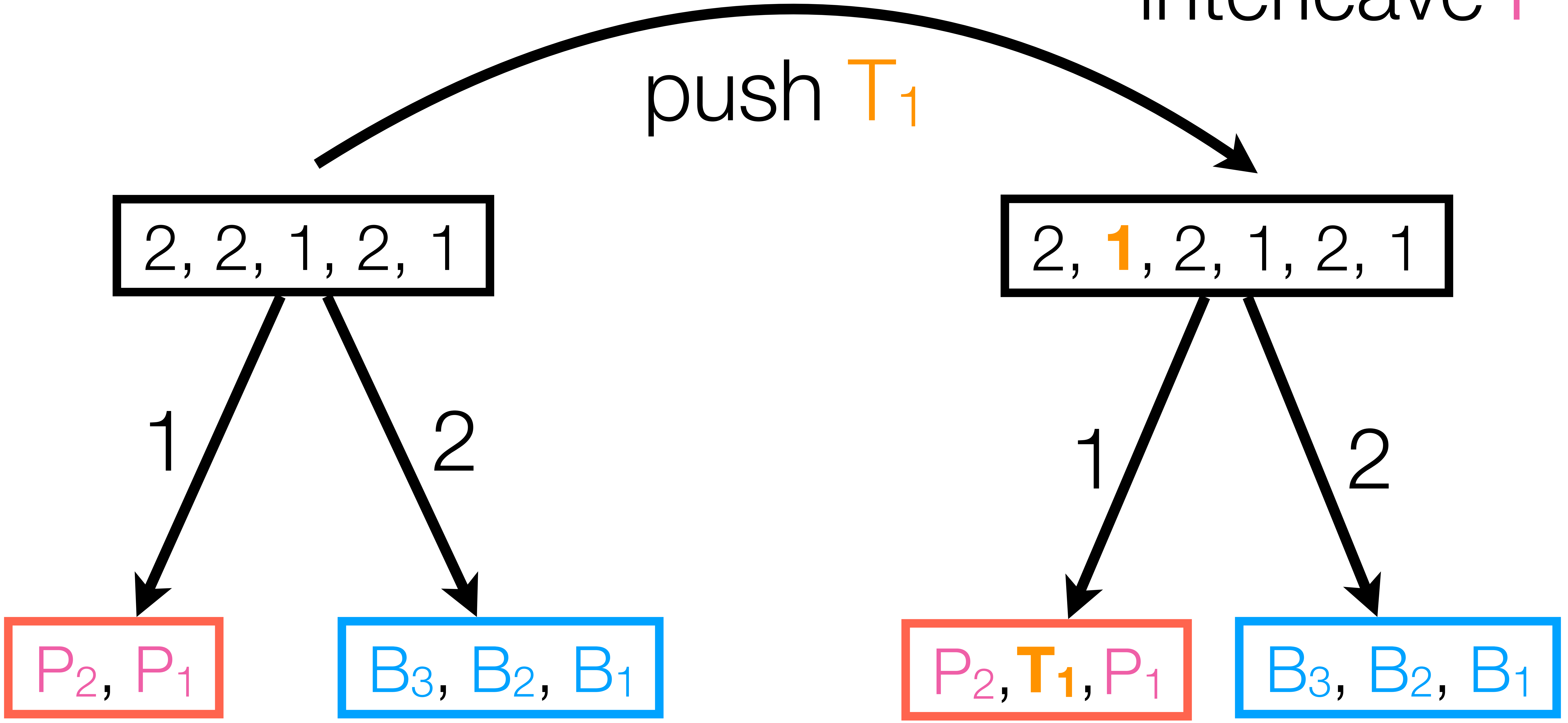

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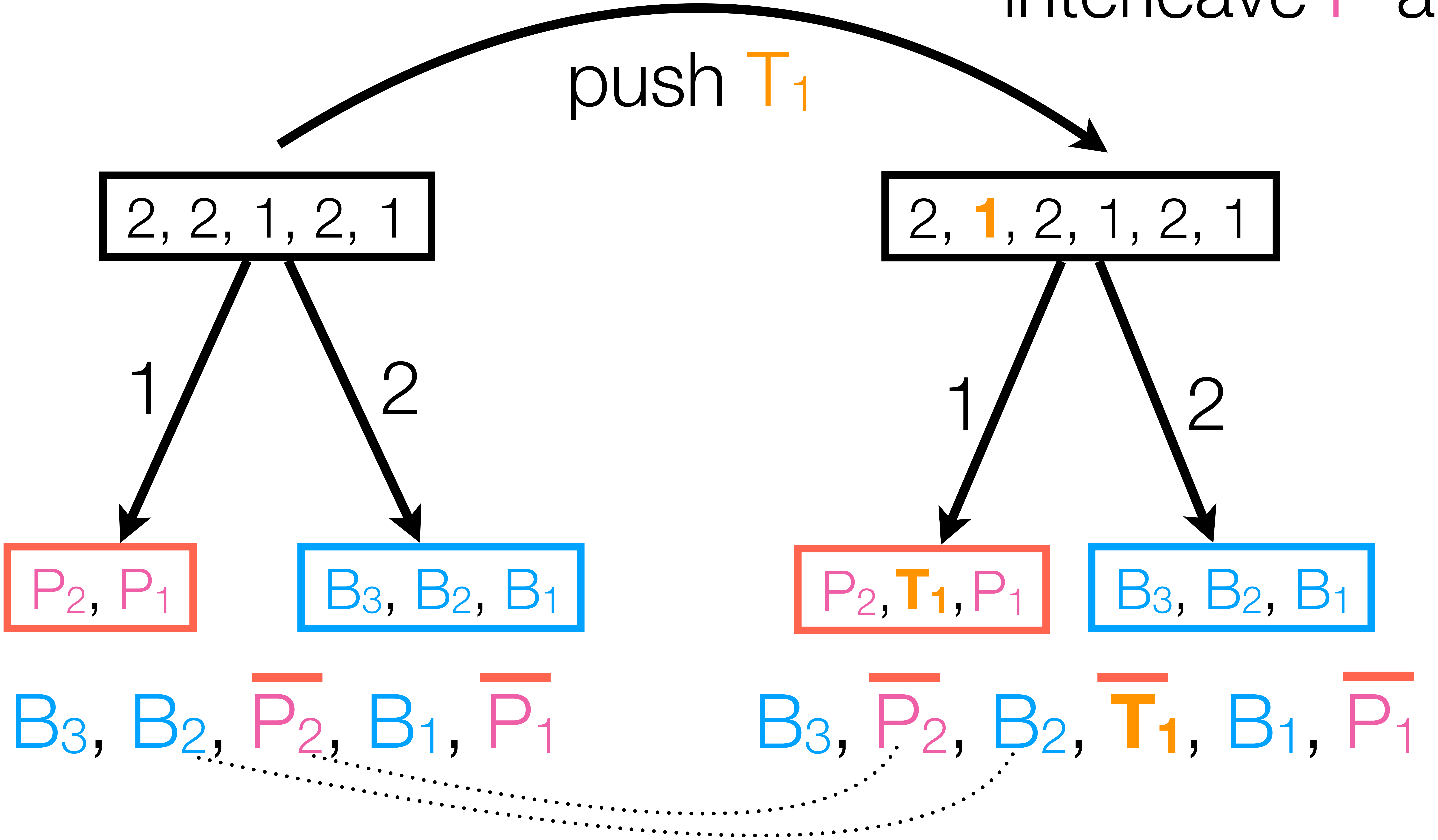

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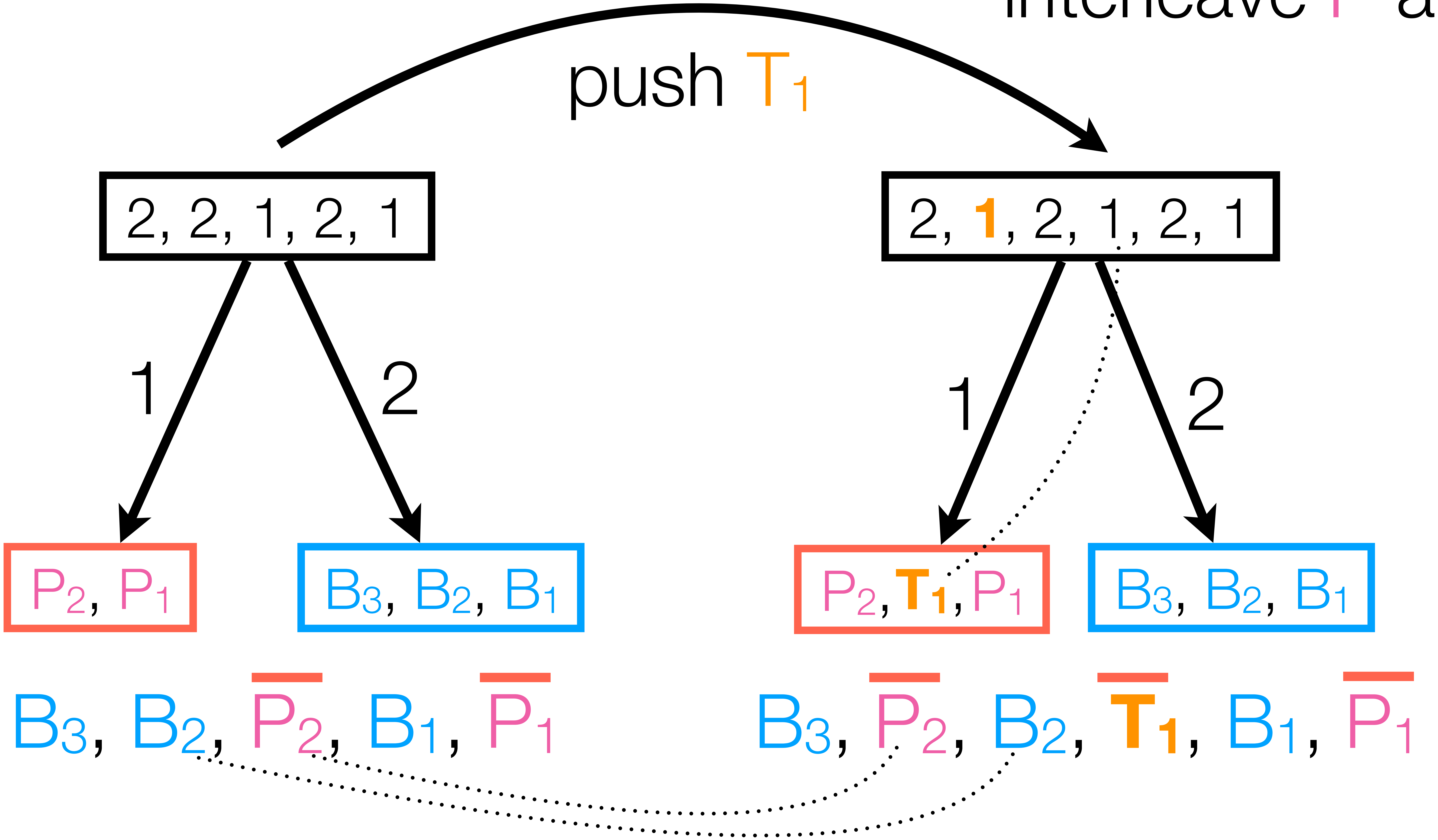

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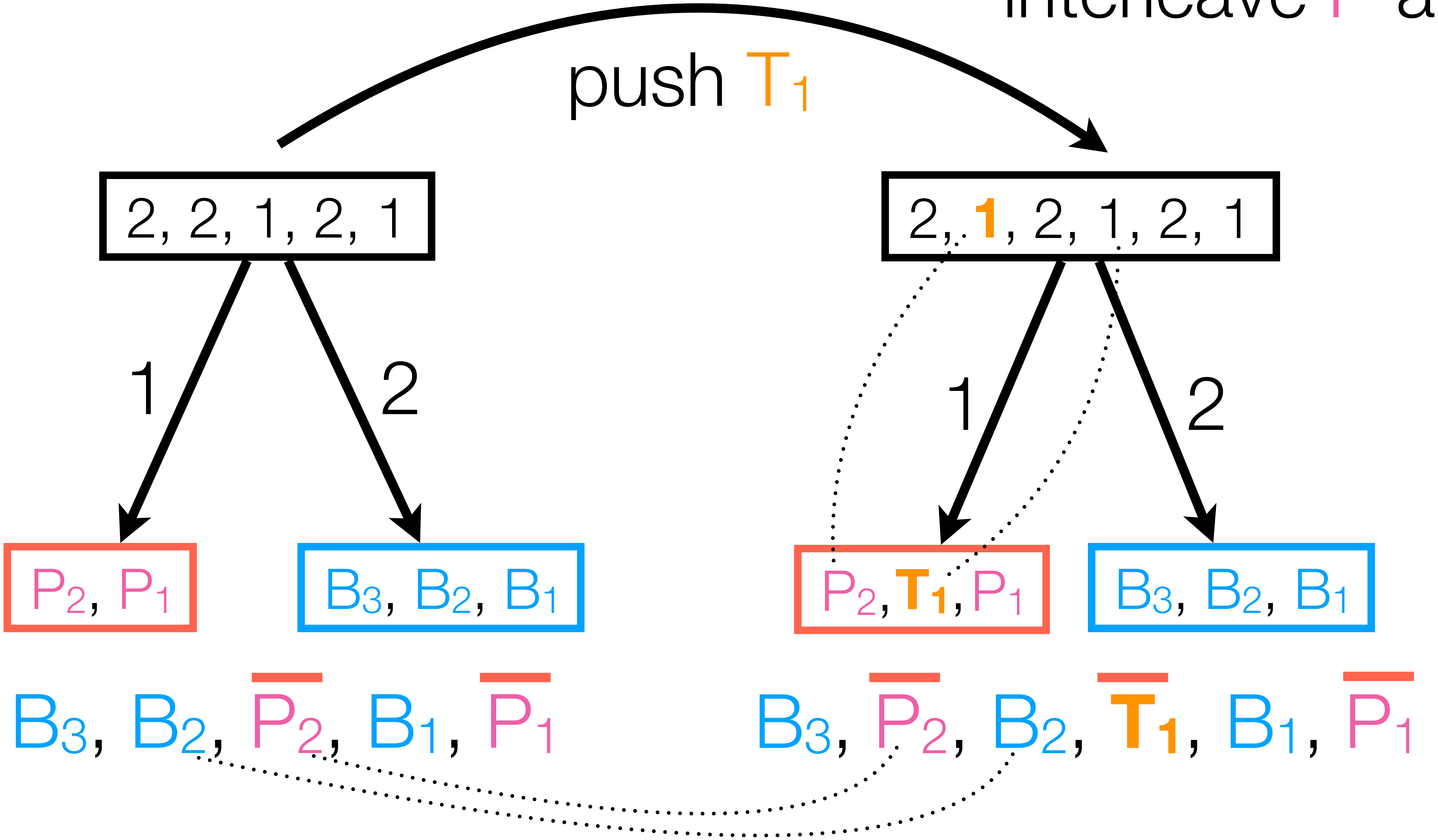

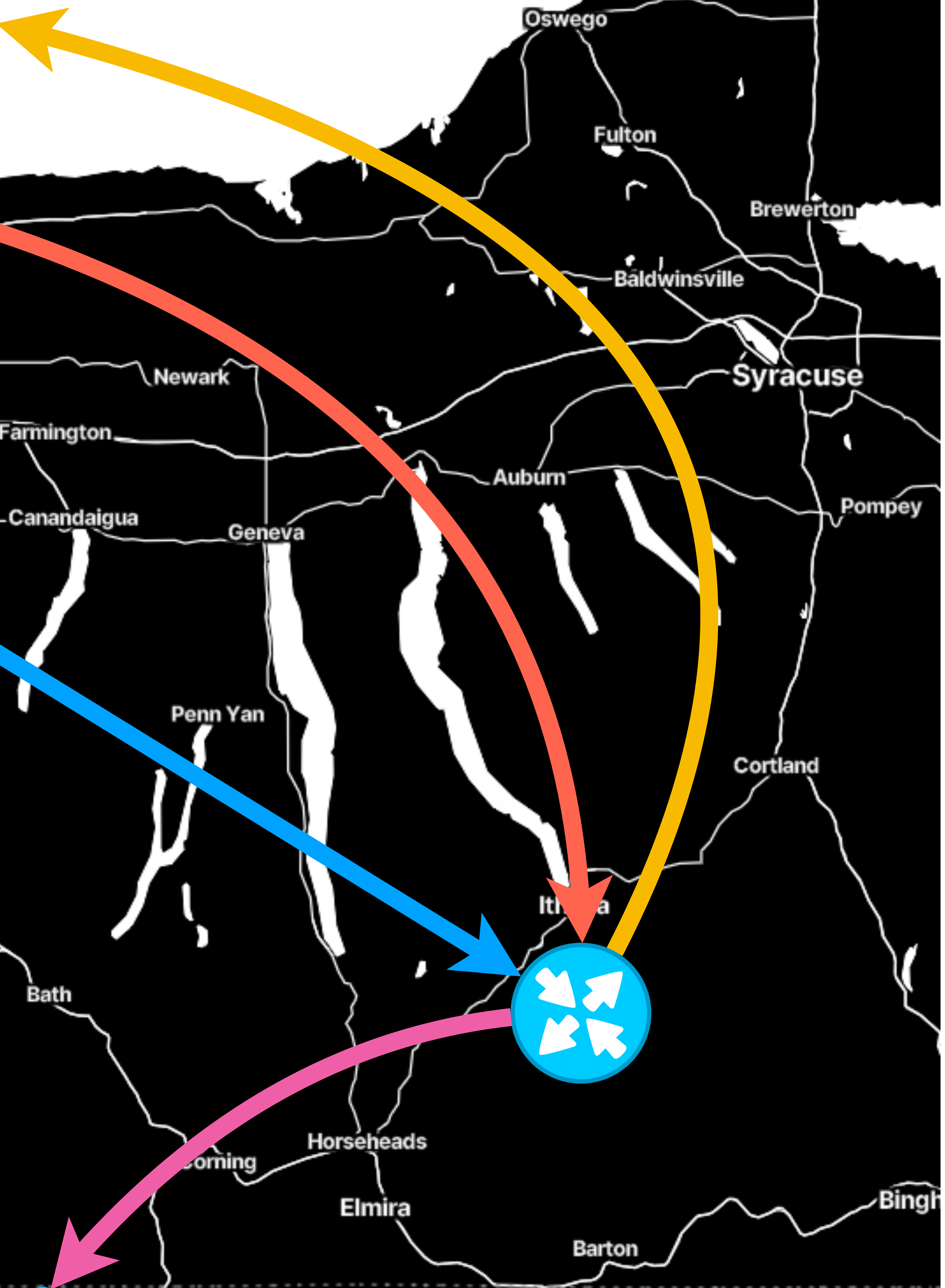

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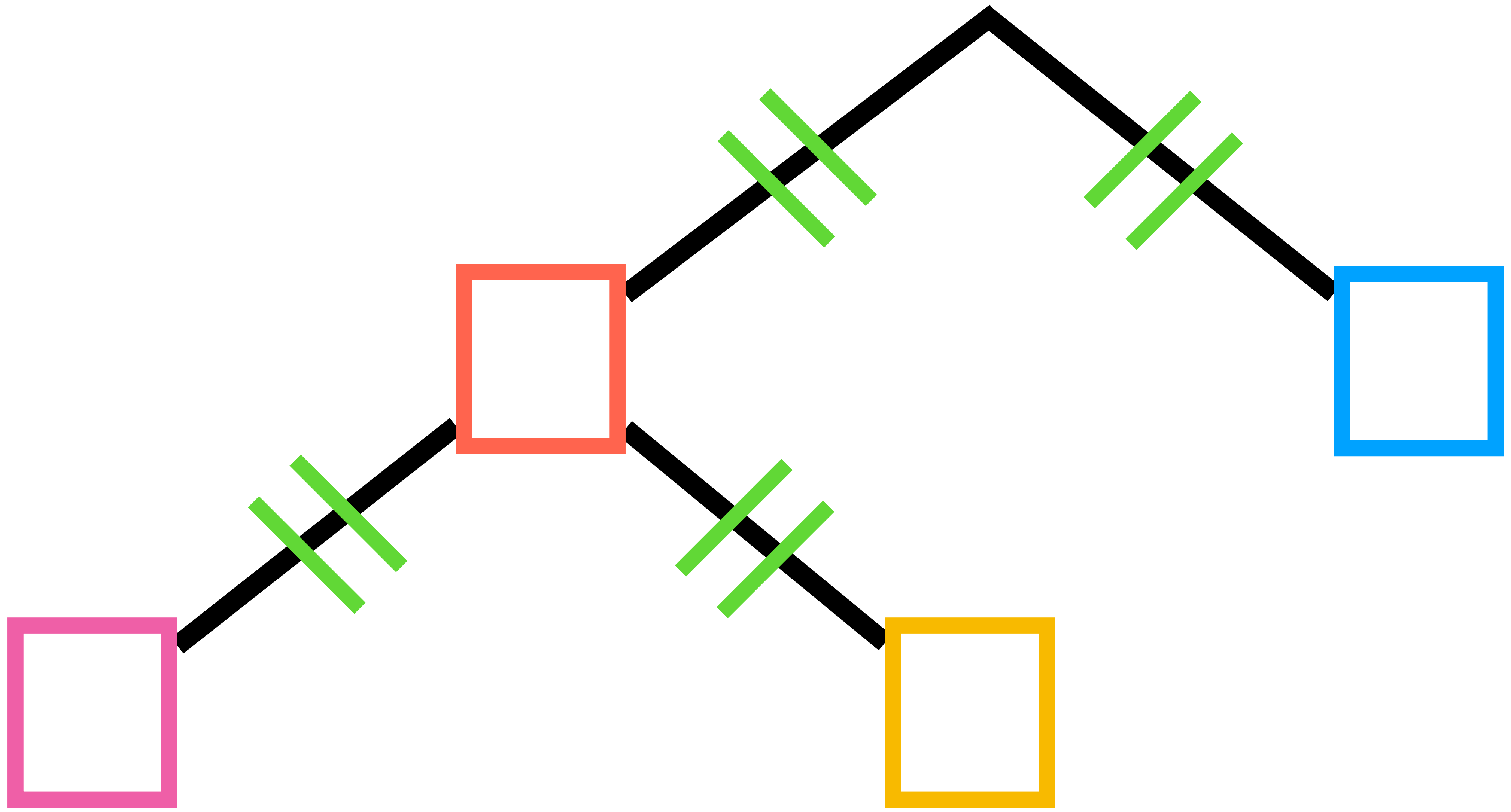

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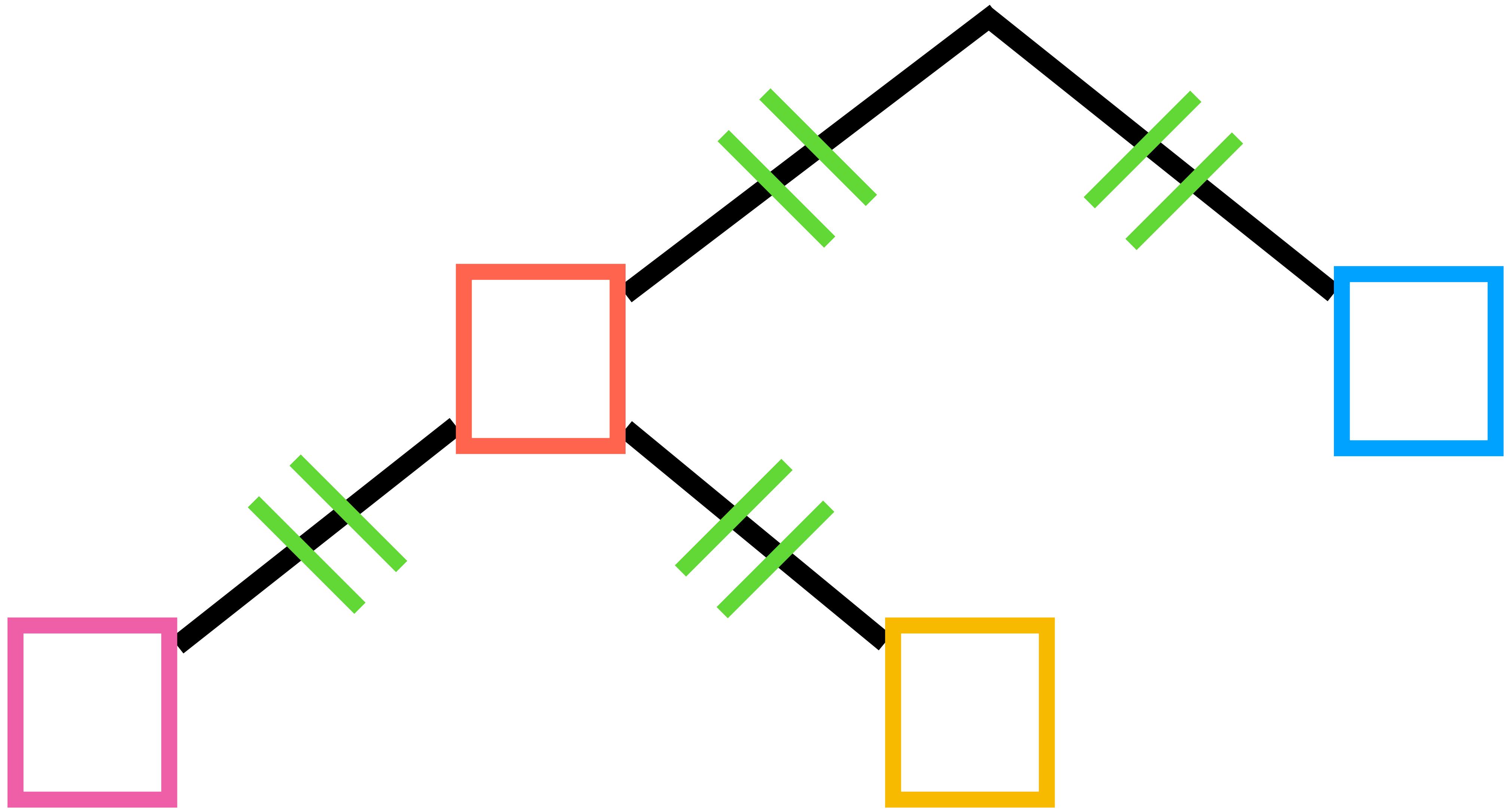
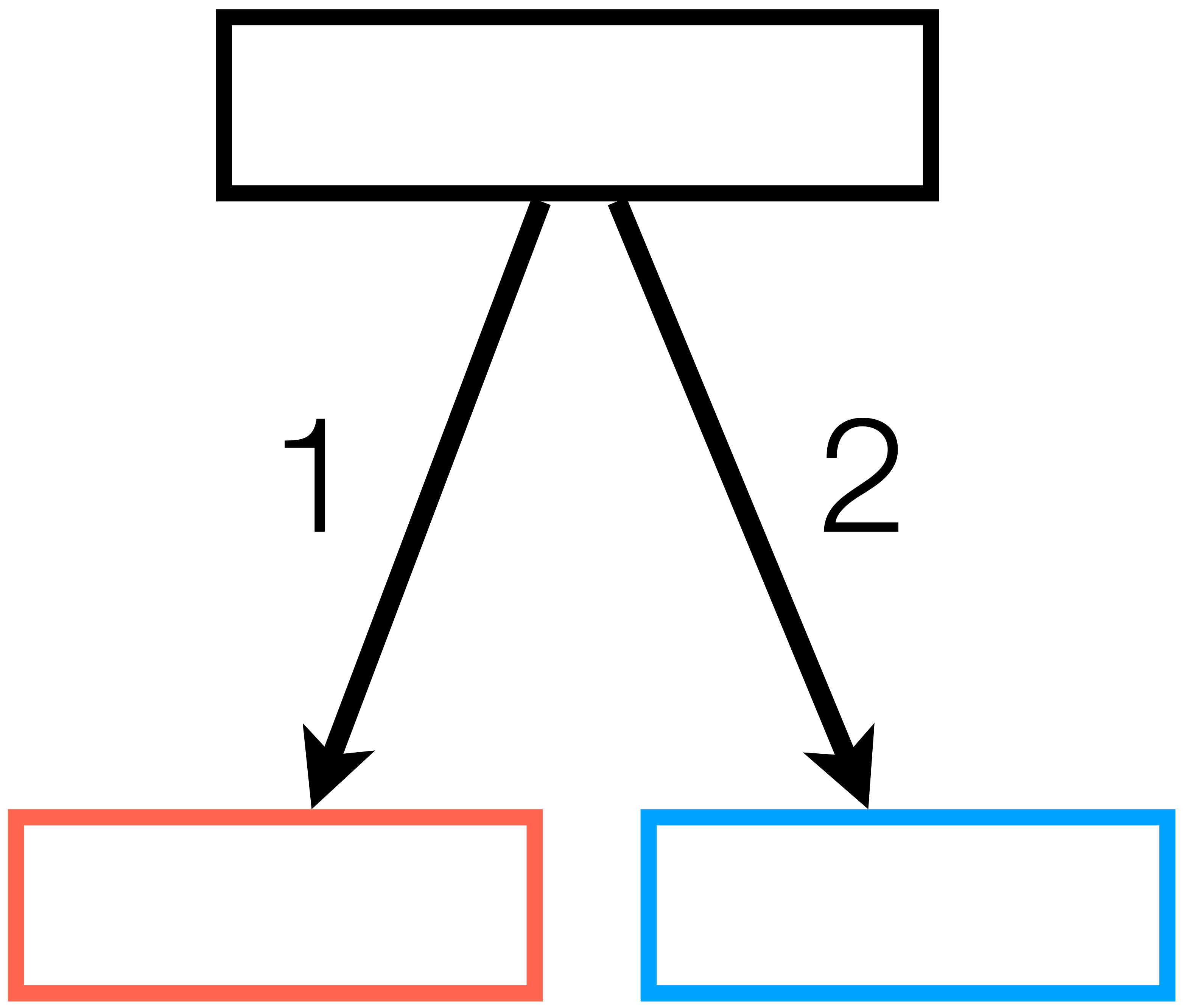


Aside: PIFO Trees

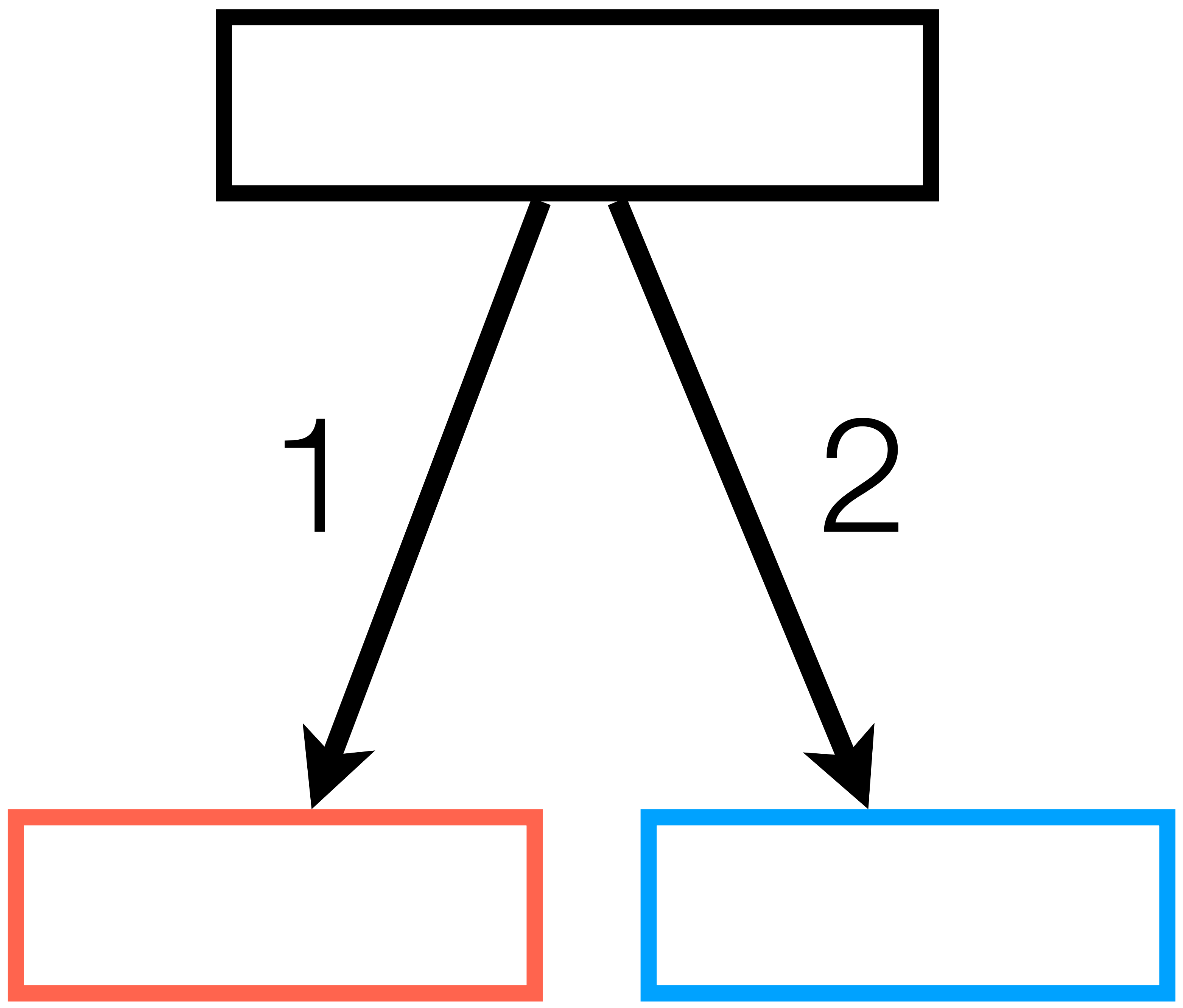
Sivaraman et al. at SIGCOMM ’16

Key Insight



Key Insight

A PIFO tree manifests a *programming language.*



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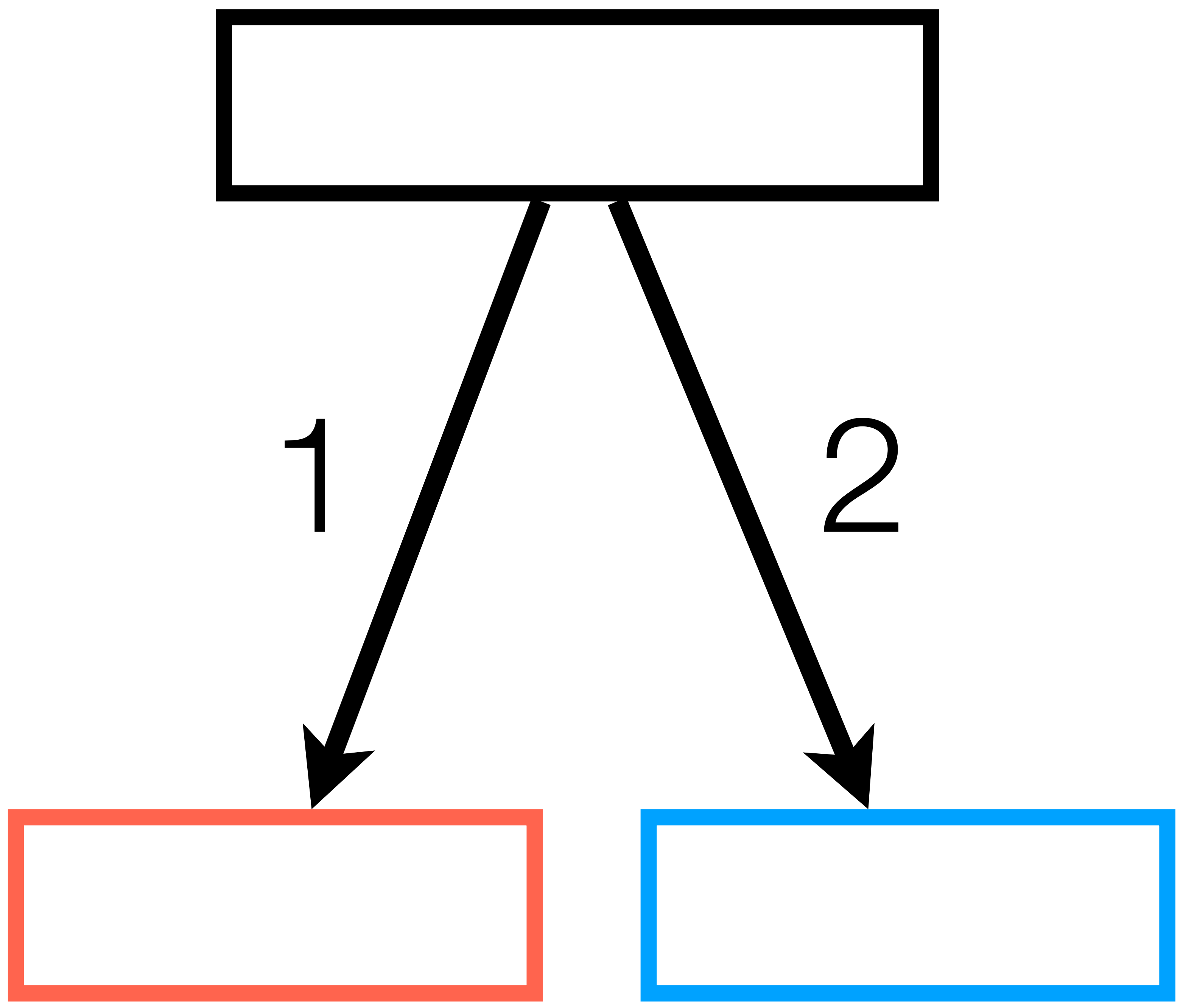
A PIFO tree manifests a *programming language.*

A program is precisely a *scheduling algorithm.*



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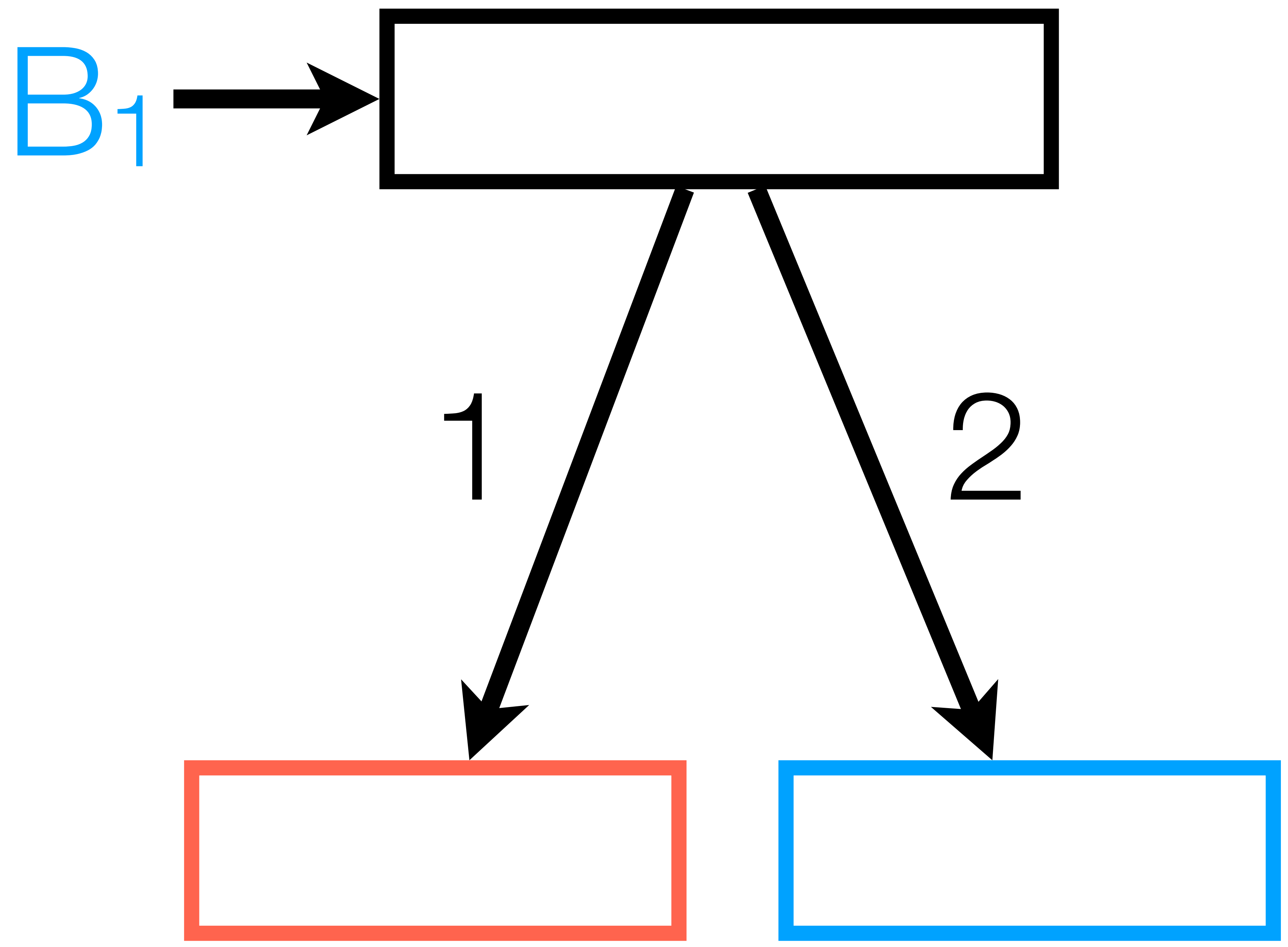

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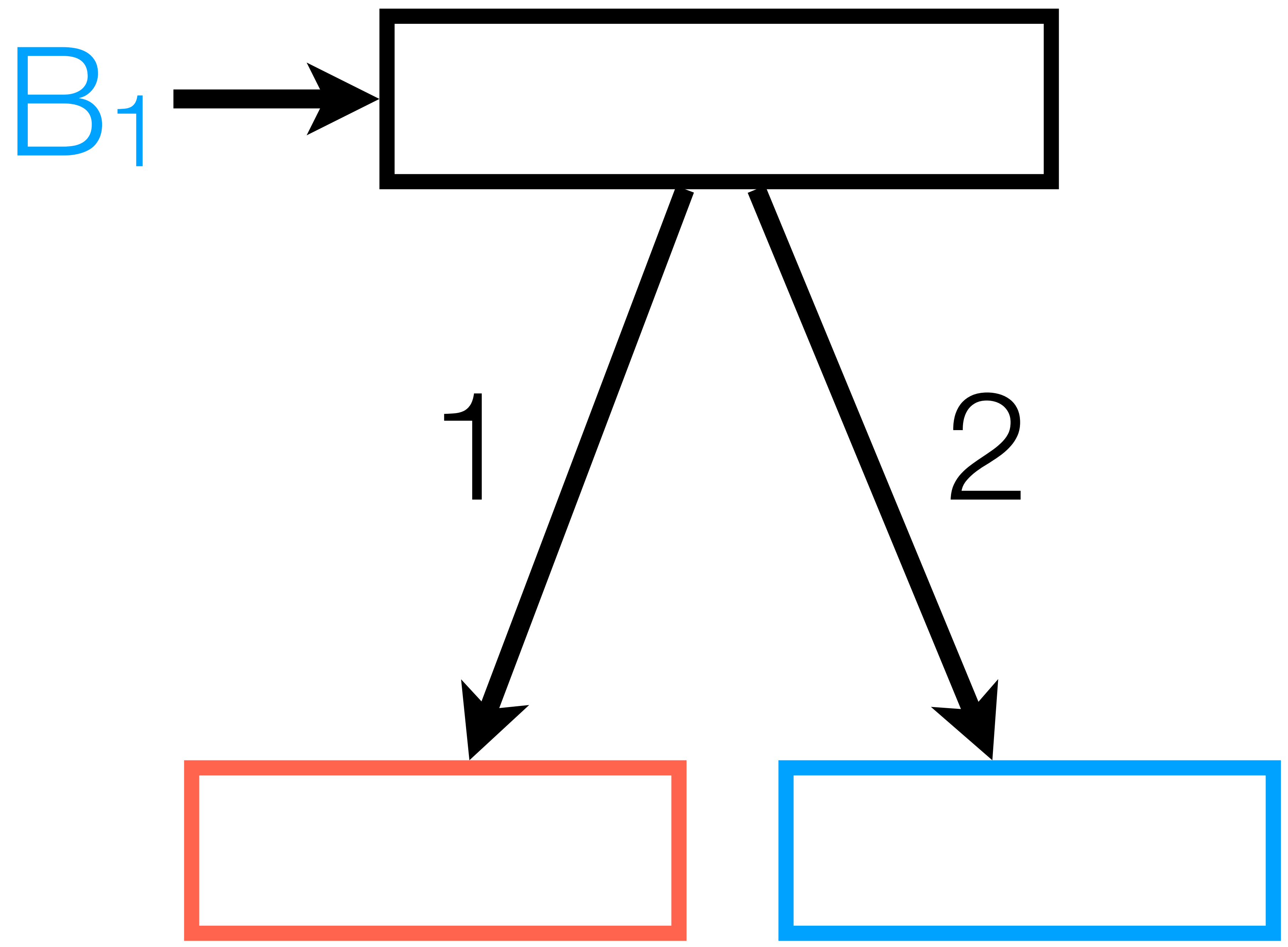
Path: $[(2, r_1), (B_1, r_2)]$



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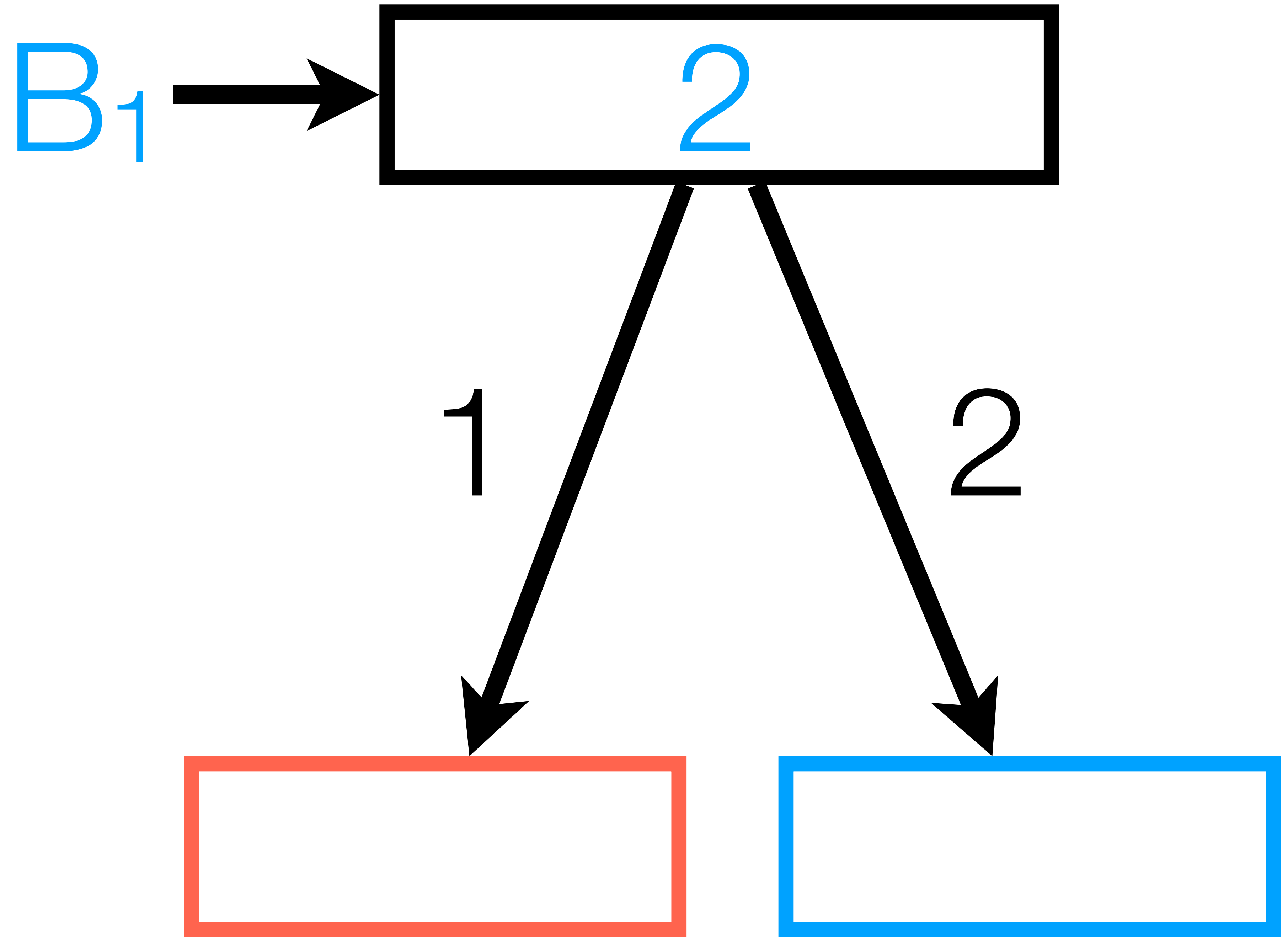

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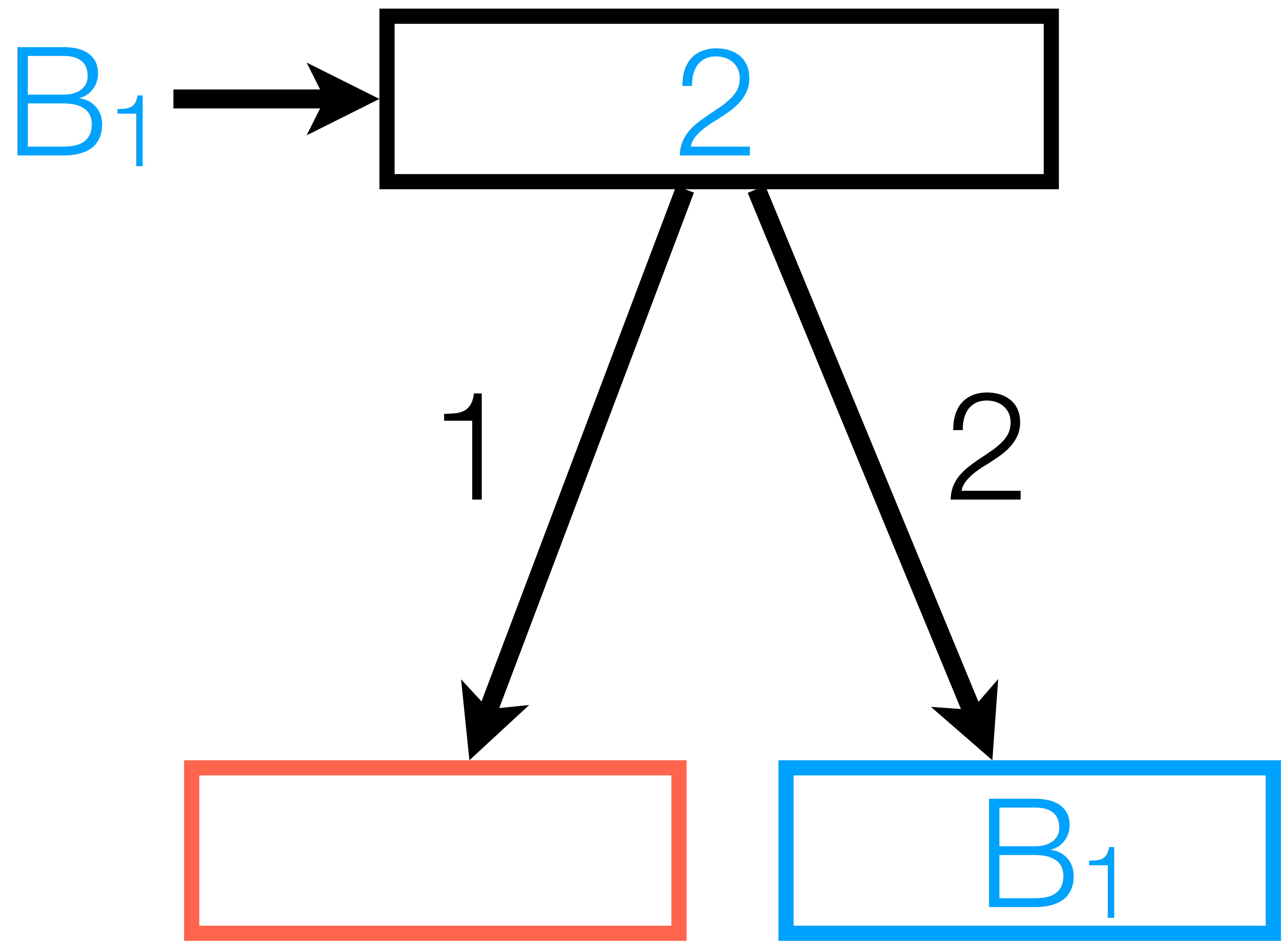

Key Insight

A PIFO tree manifests a *programming language.*

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tree shape ⟷ language expressivity

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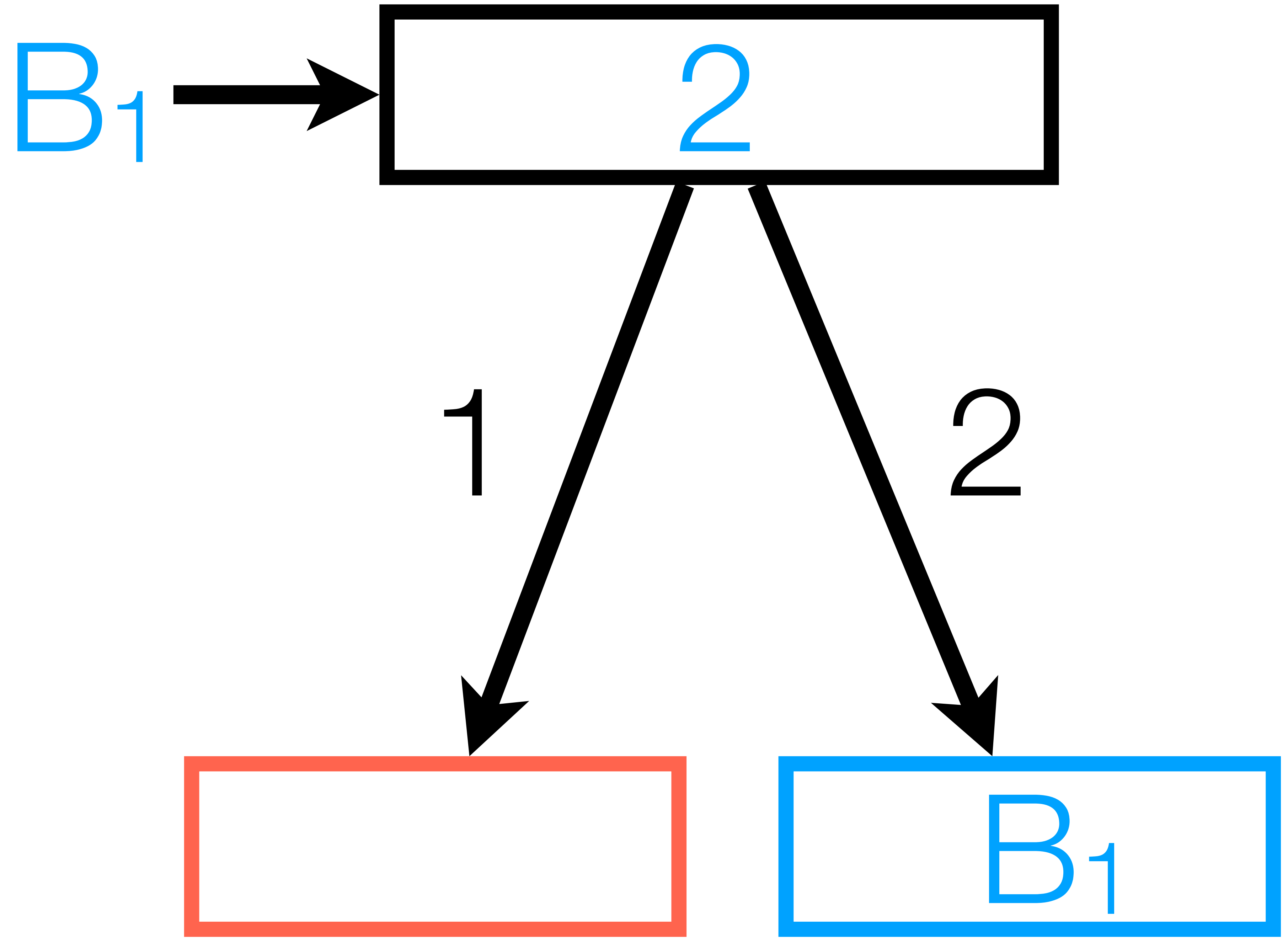

Which leads to some very PL-ey questions:

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Compare expressivity of languages?

Which leads to some very PL-ey questions:

tree shape $\longleftrightarrow$ language expressivity

Compare expressivity of languages?
Compare expressivity of *trees*?

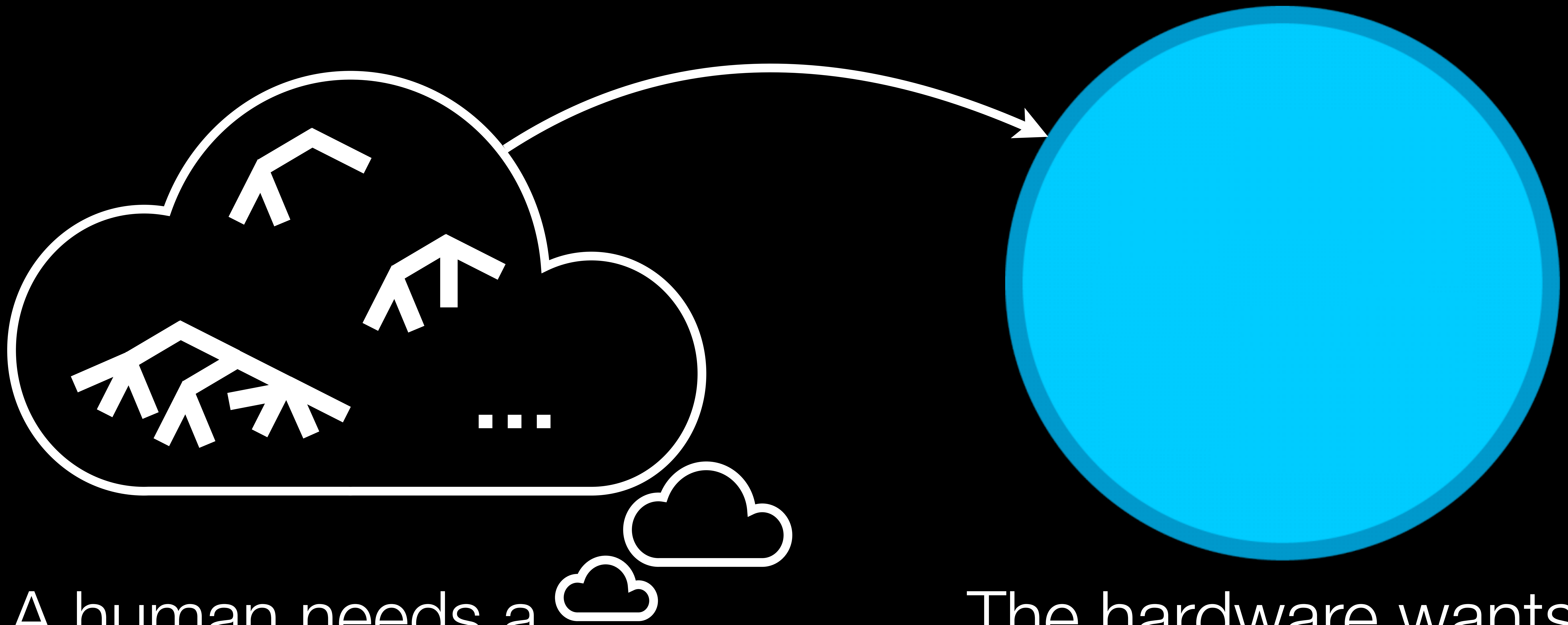
Which leads to some very PL-ey questions:

tree shape $\longleftrightarrow$ language expressivity

Compare expressivity of languages?
Compare expressivity of *trees*?

Compile a program so it runs against a new tree?

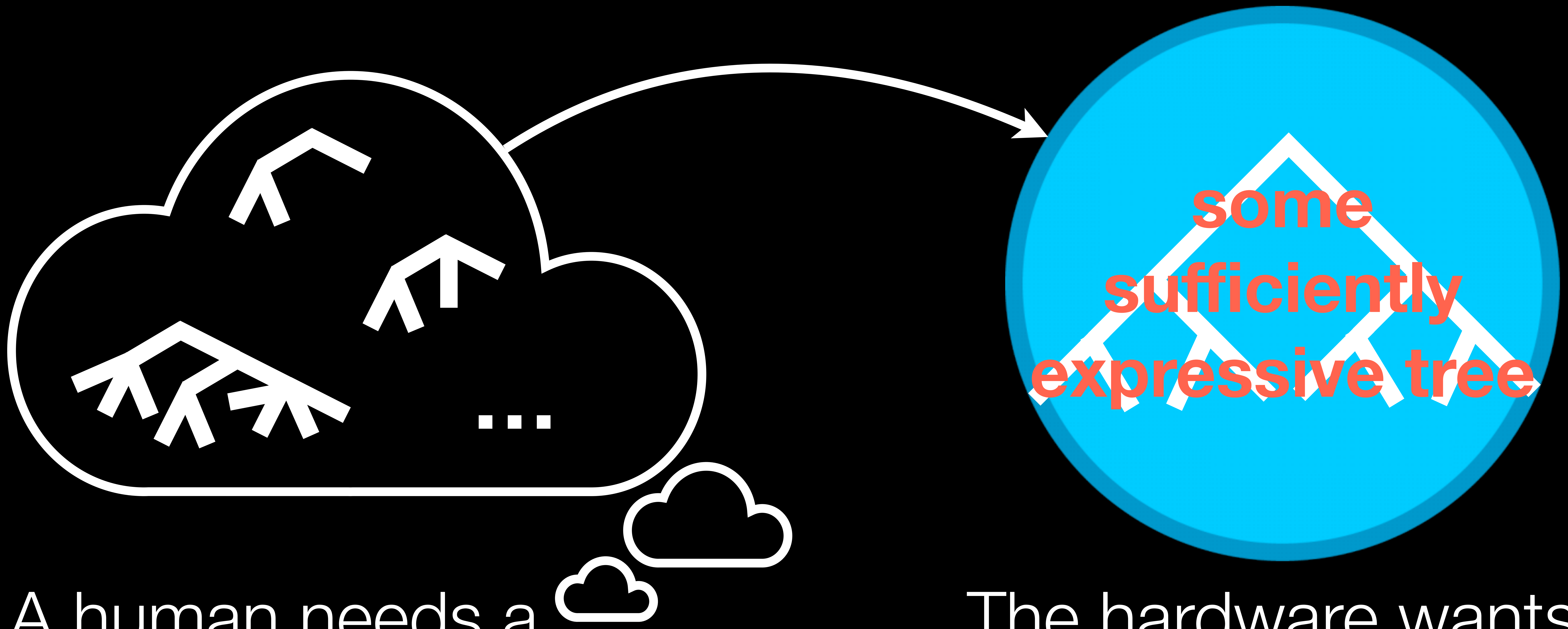
~~No general way to deploy our gadget.~~



A human needs a *range* of trees.

The hardware wants to support *one* tree.

~~No general way to deploy our gadget.~~



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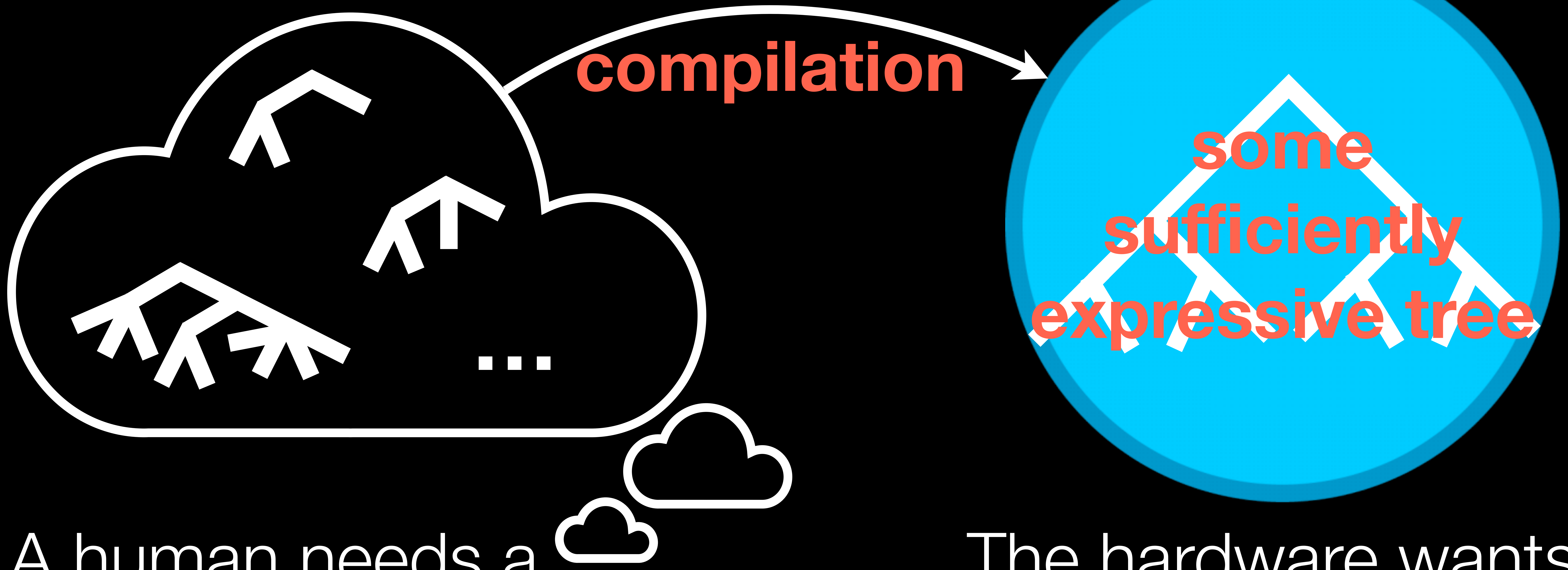

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Contributions

Contributions

Formal model of PIFO trees

Contributions

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General theorems of expressiveness w.r.t. tree shape

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Formal model of PIFO trees

General theorems of expressiveness w.r.t. tree shape

Compiler

Contributions

Formal model of PIFO trees

General theorems of expressiveness w.r.t. tree shape

Compiler

Simulator

Expressivity of trees

Trees with more leaves are more expressive.

Taller trees are more expressive.

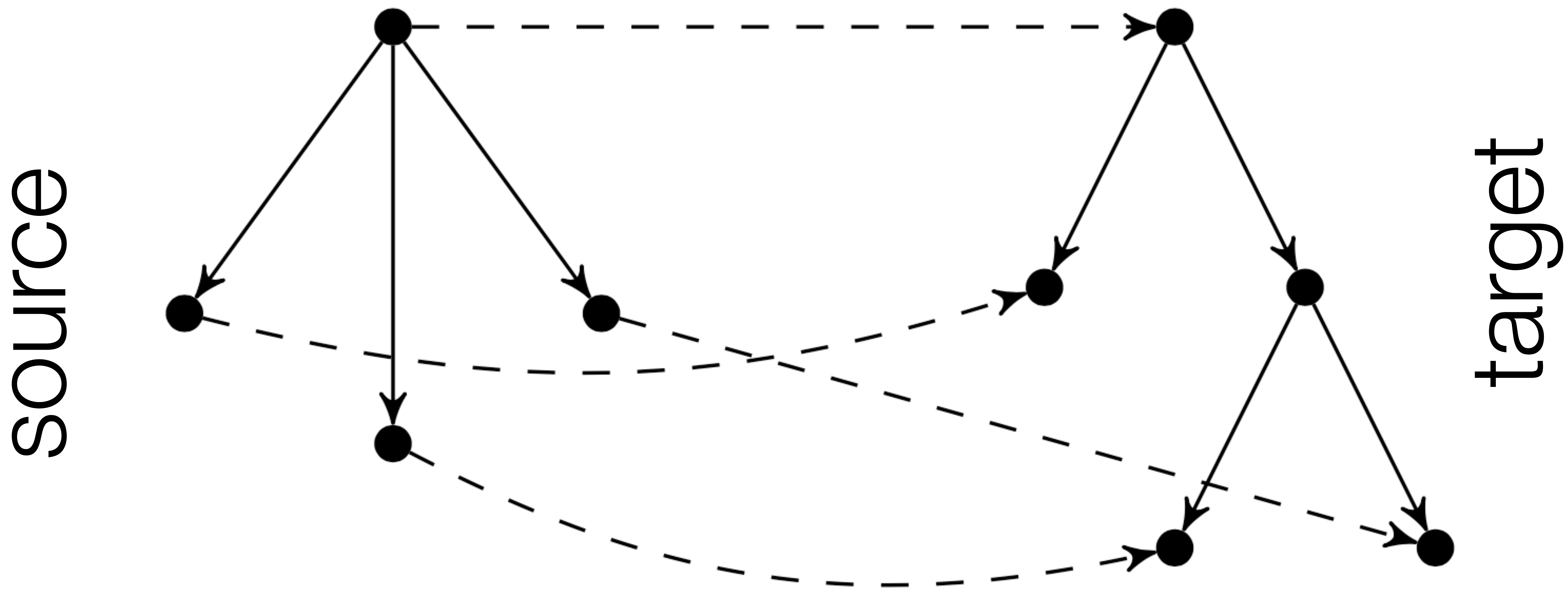
Expressivity of trees

Trees with more leaves are more expressive.
Taller trees are more expressive.

Captured elegantly by:

Homomorphic embedding.
Map root to root, leaves to leaves. Respect ancestry.

Expressivity of trees



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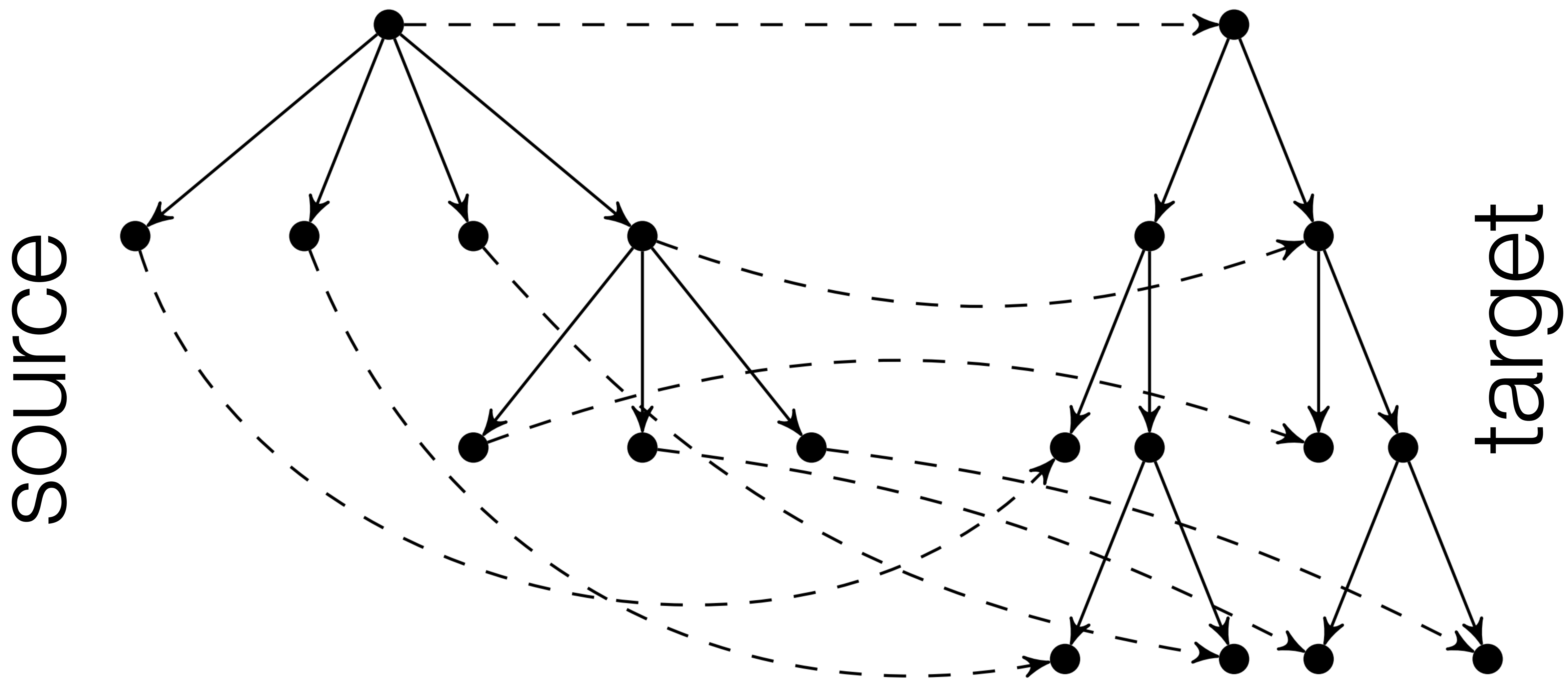
Expressivity of trees



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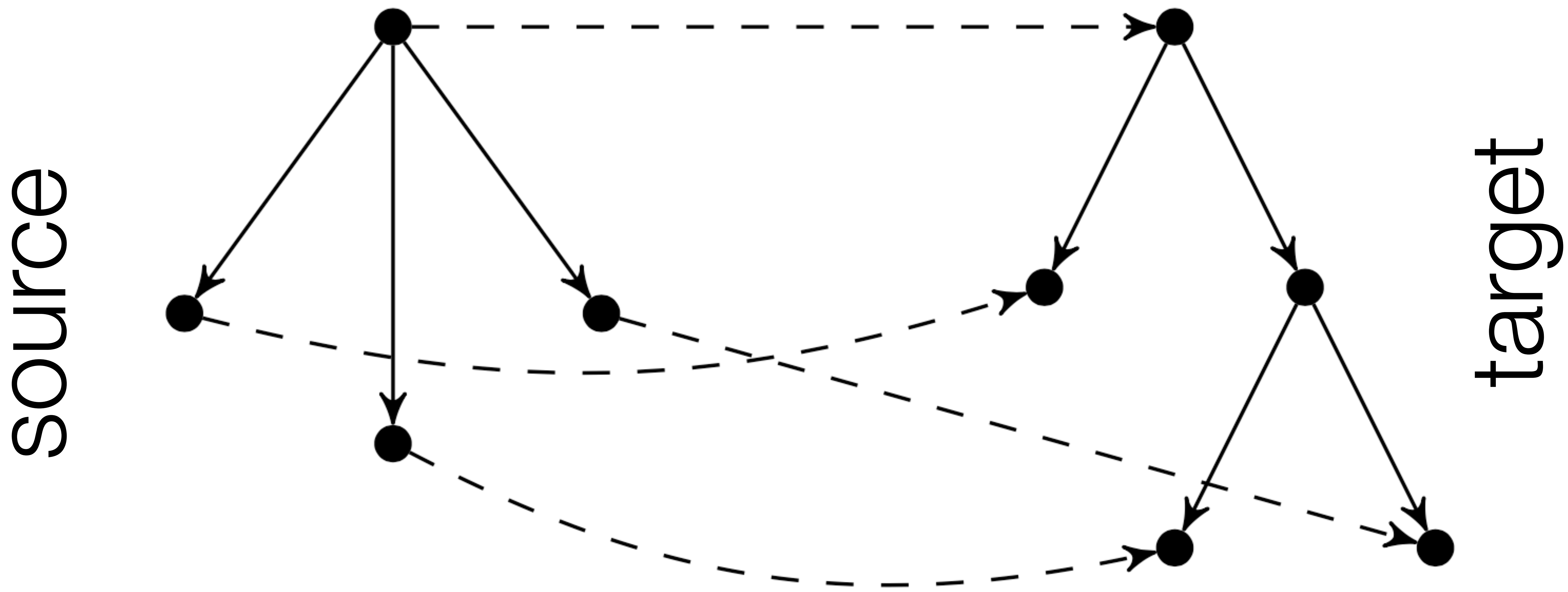

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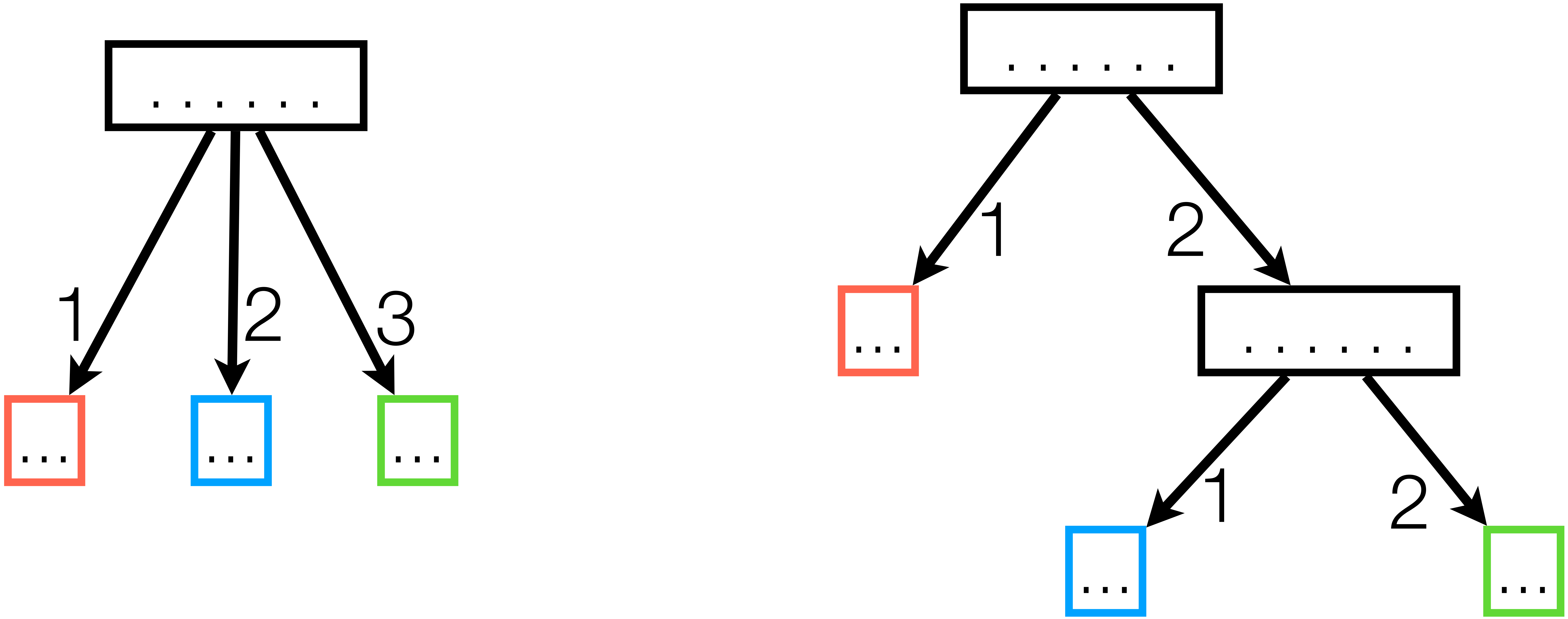
Compiling programs



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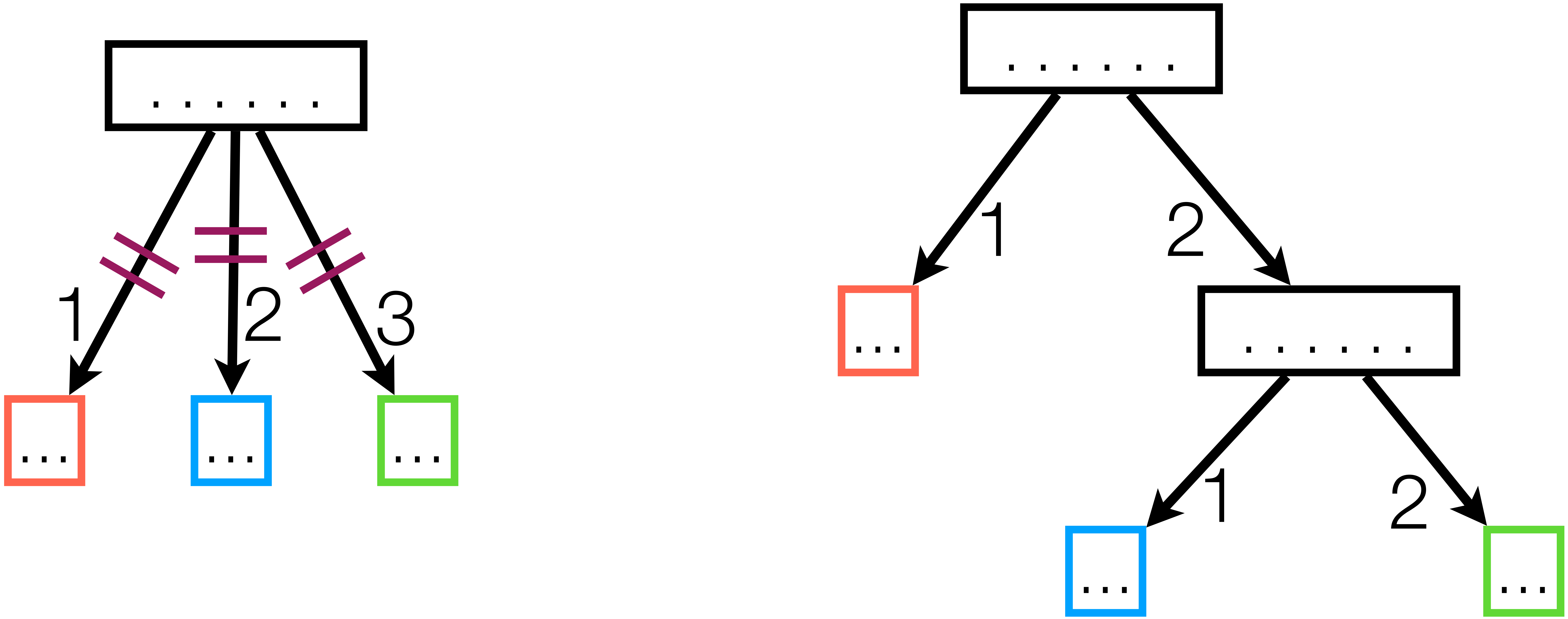

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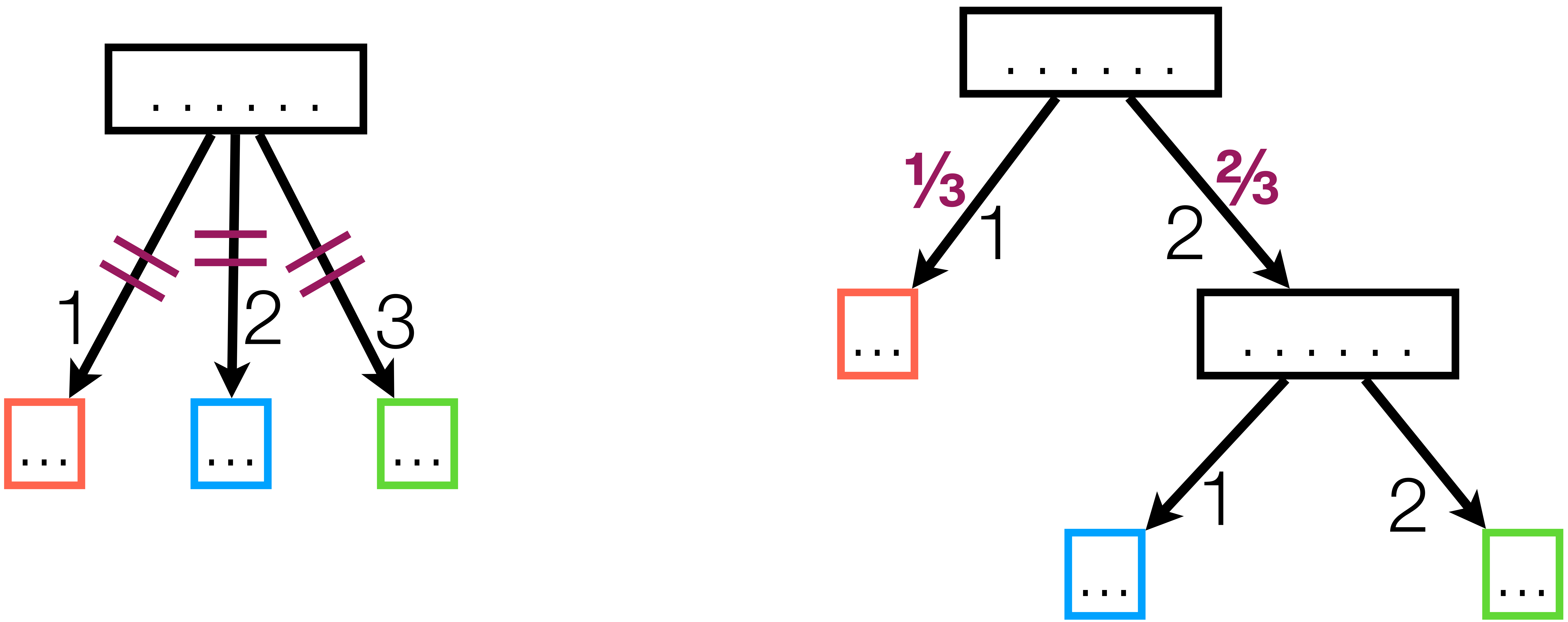

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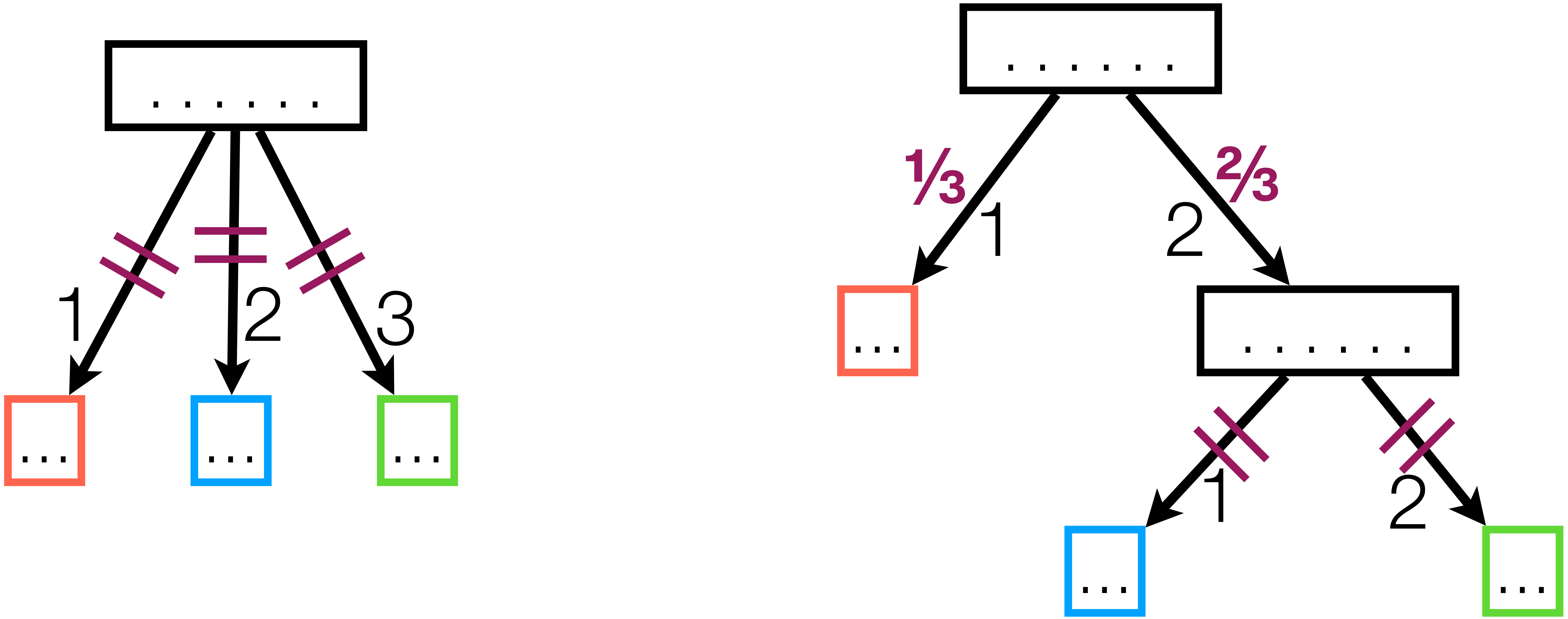

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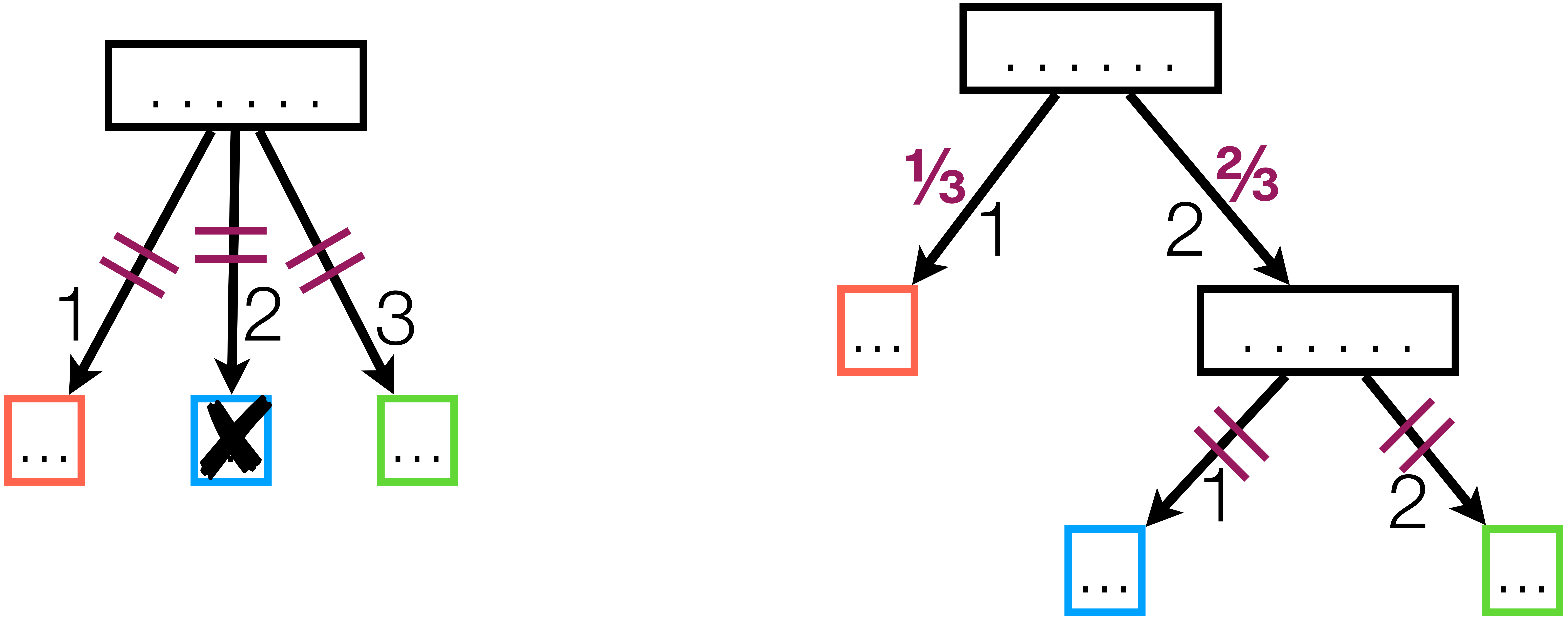

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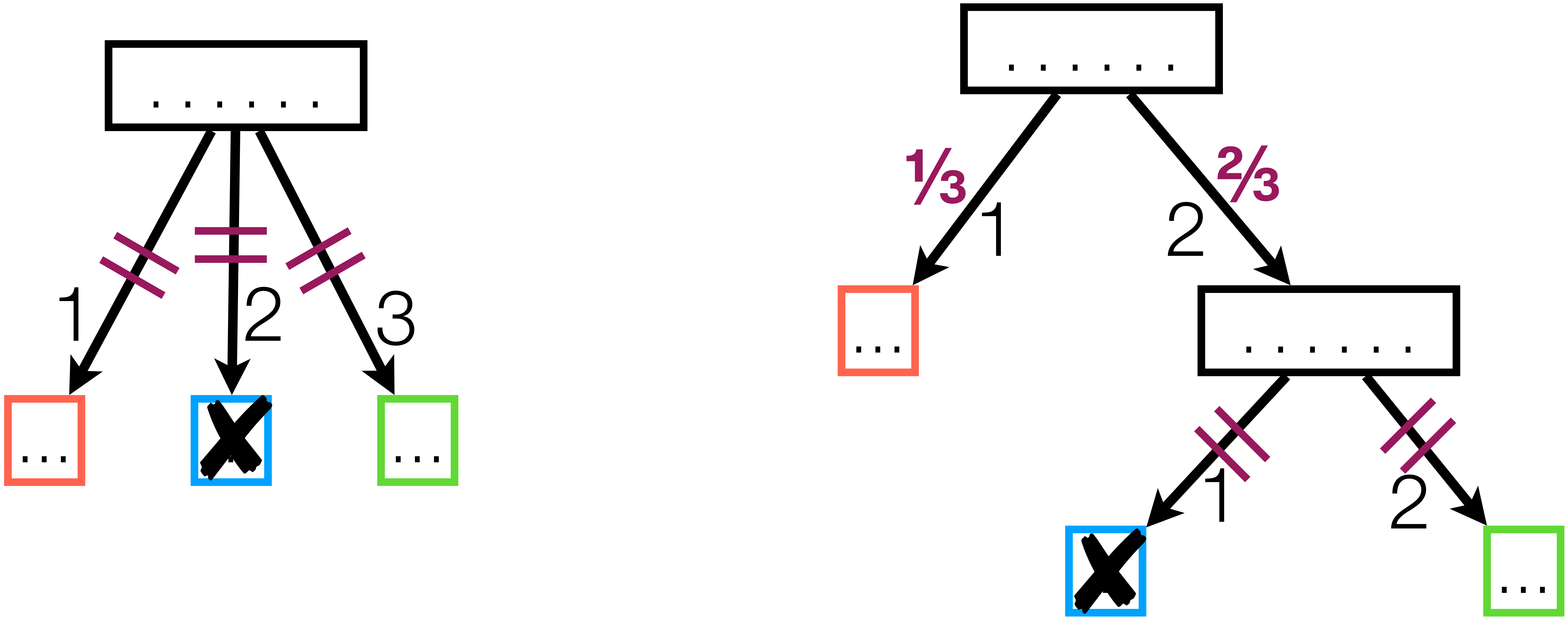
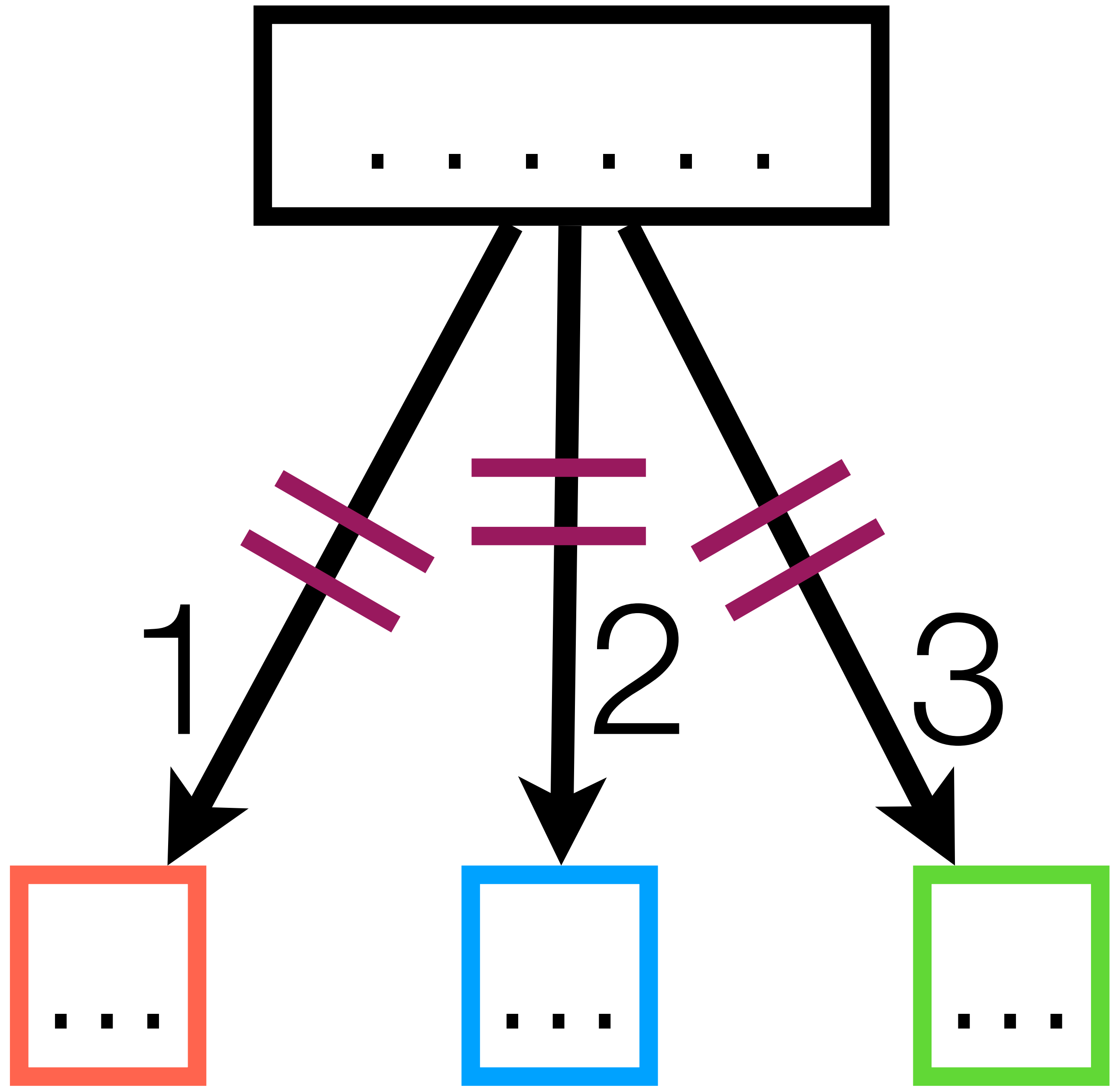
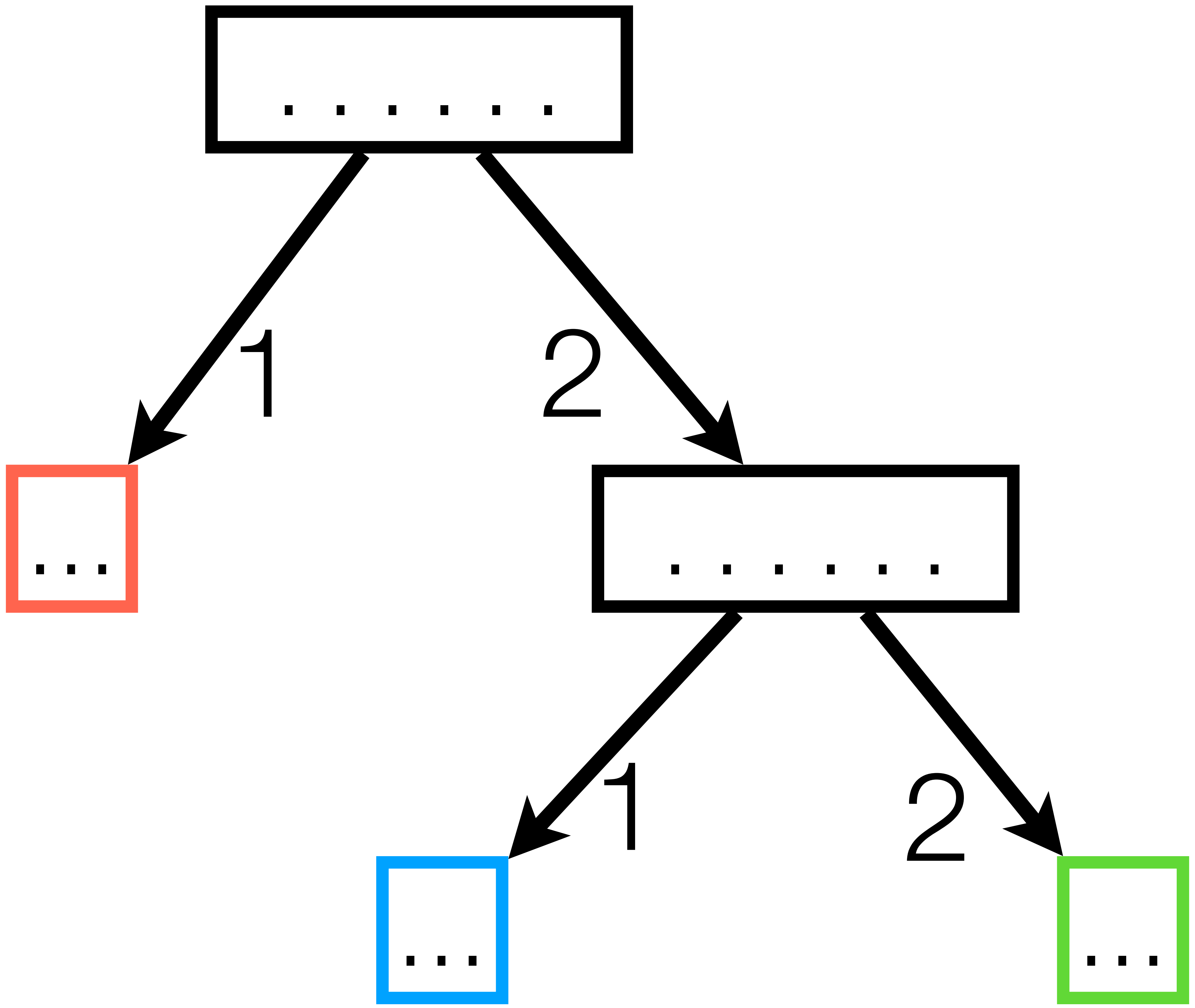


Compiling programs





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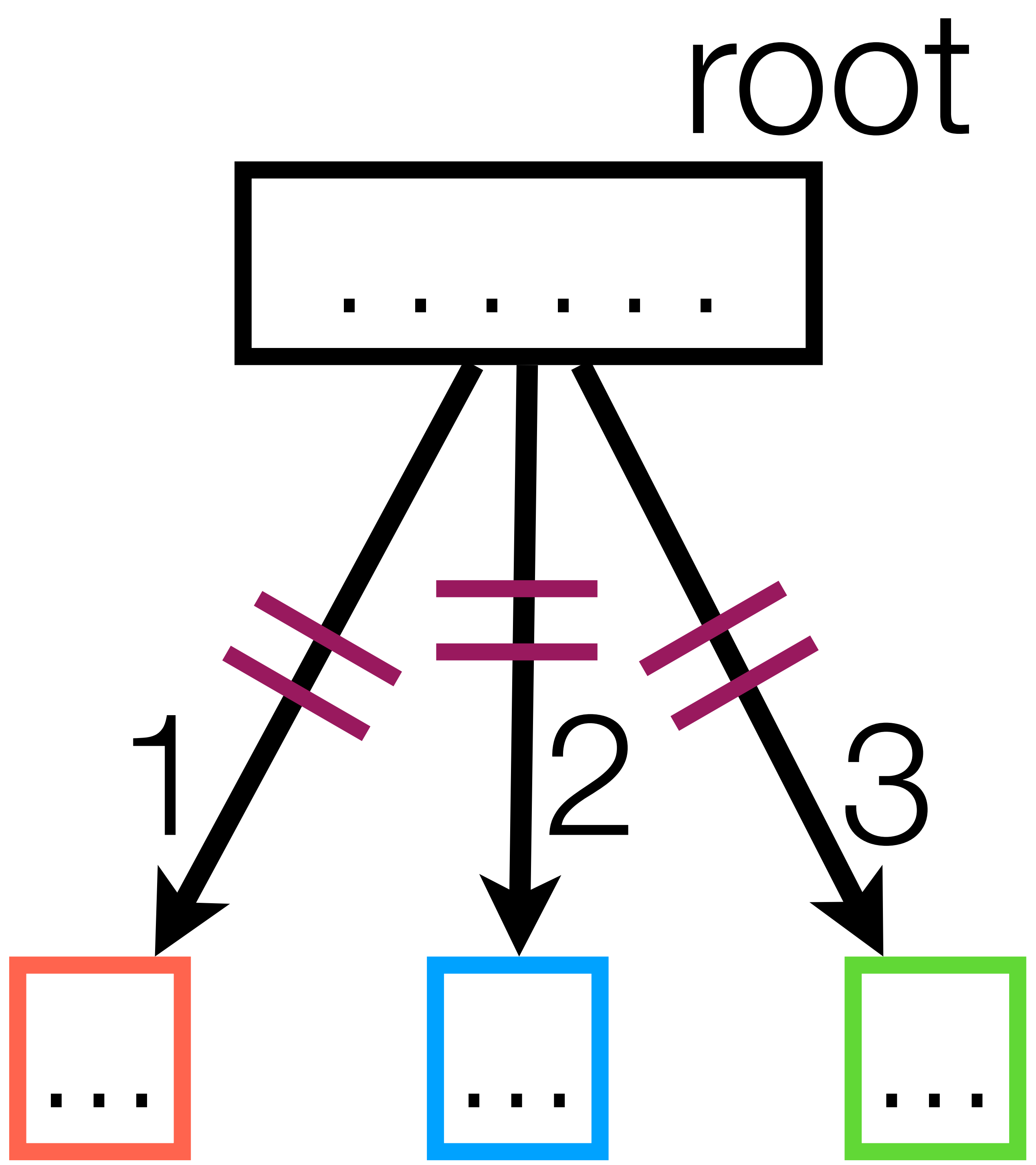

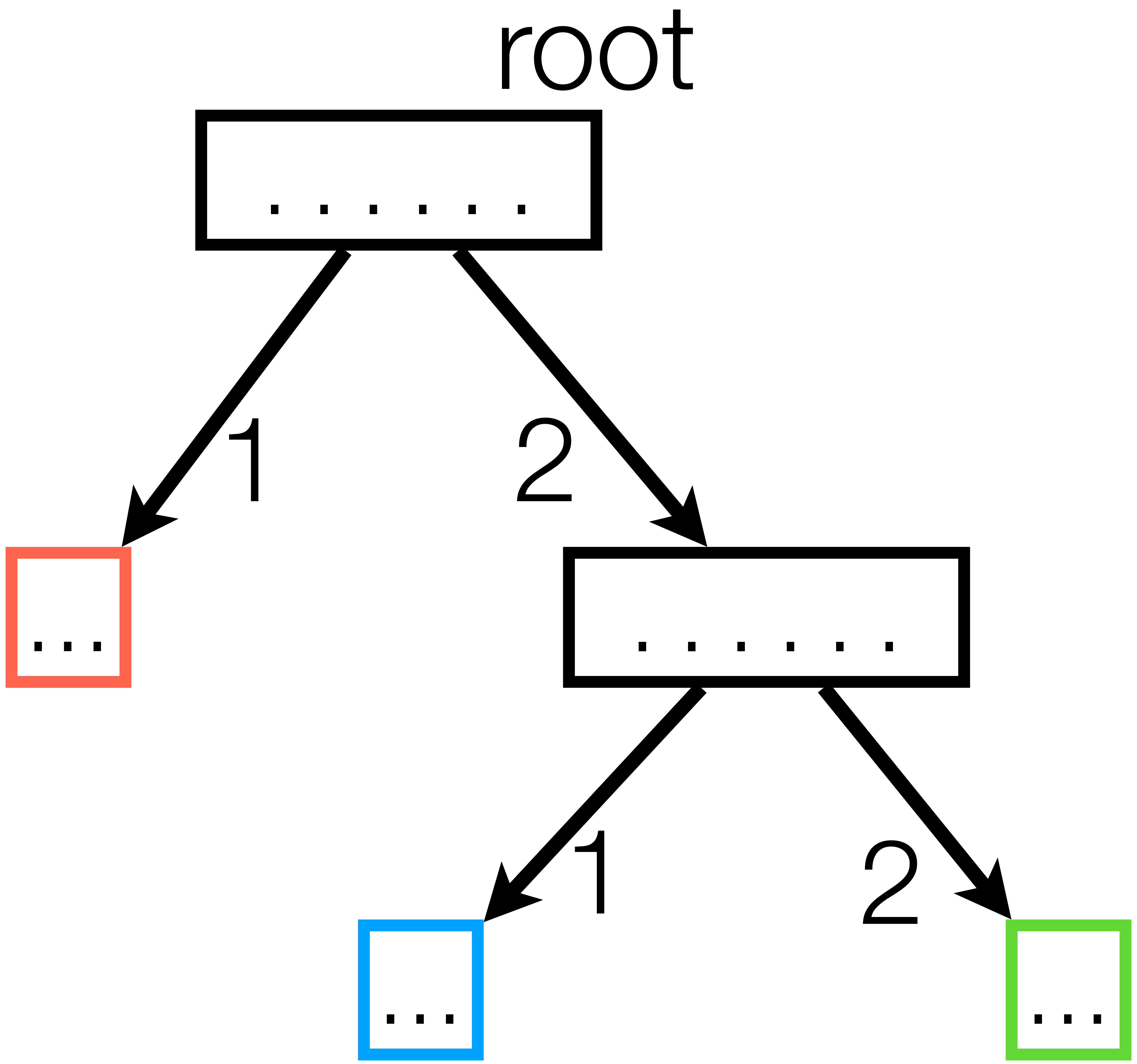

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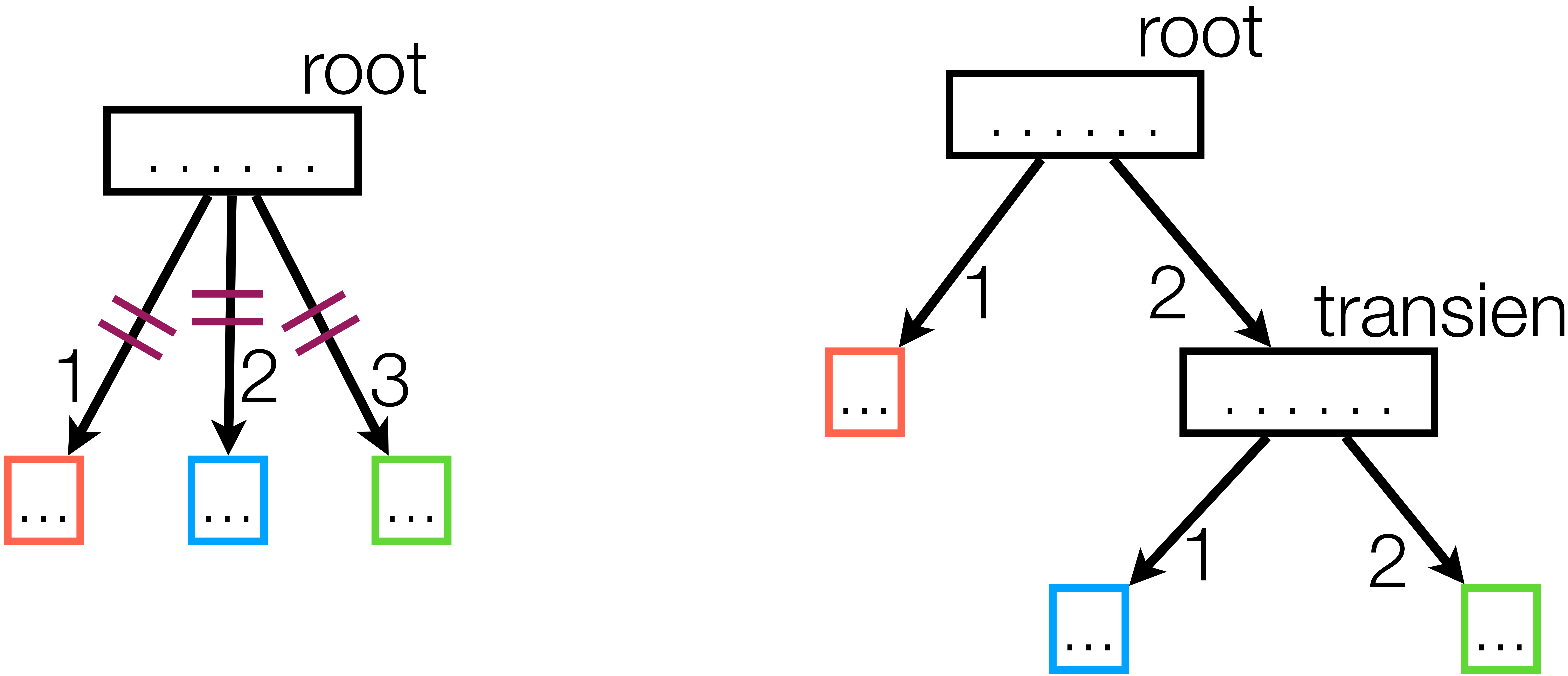

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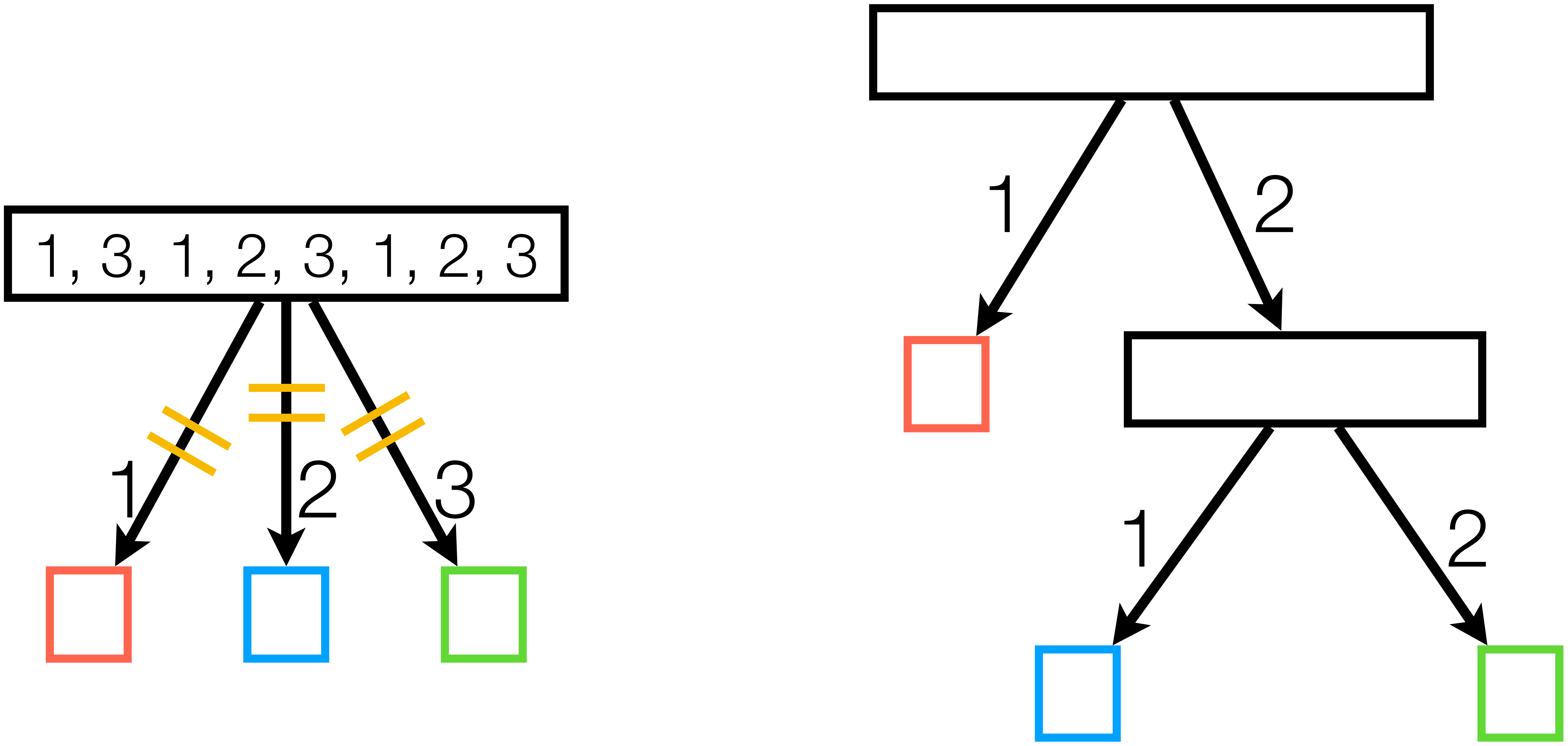

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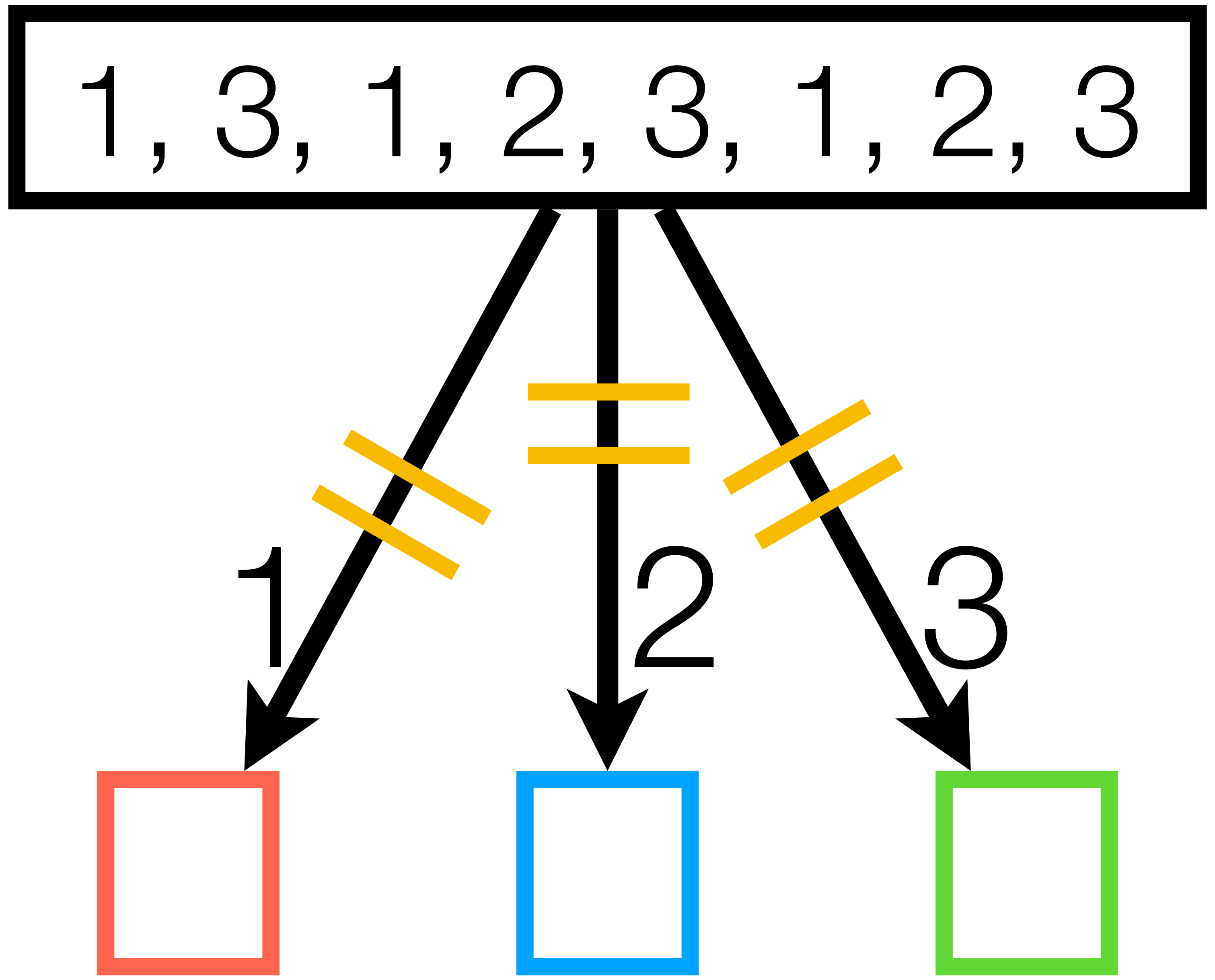

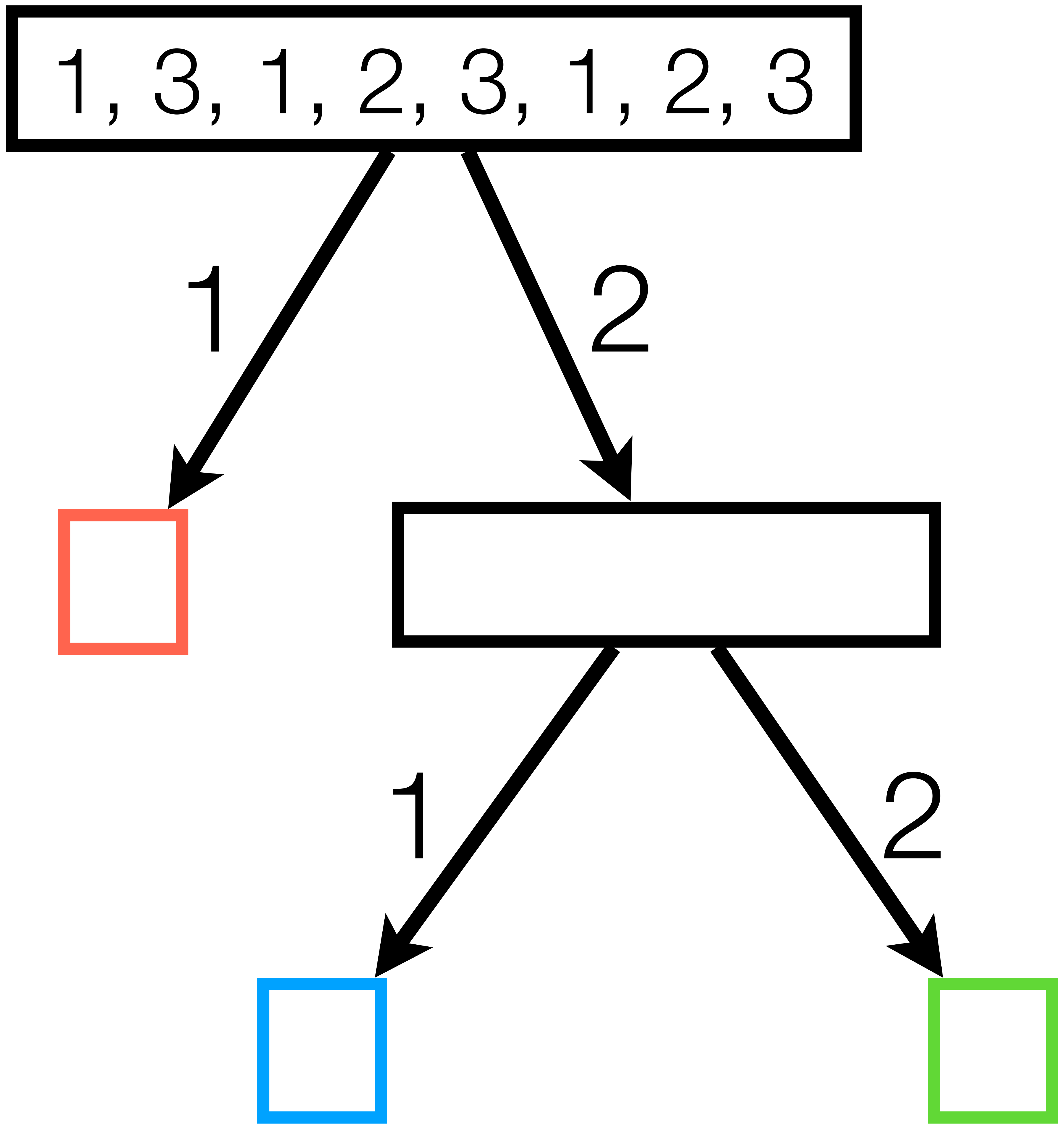

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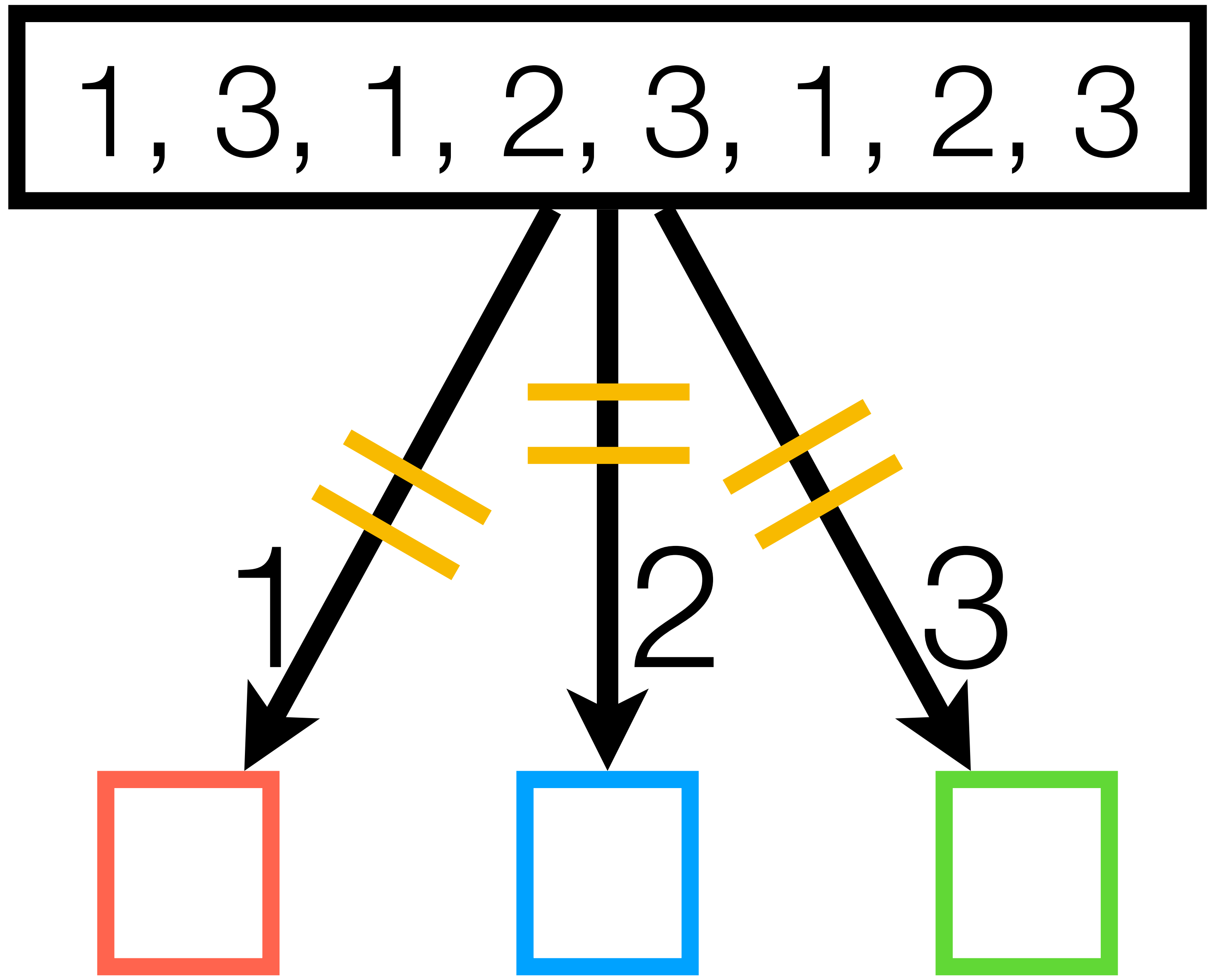

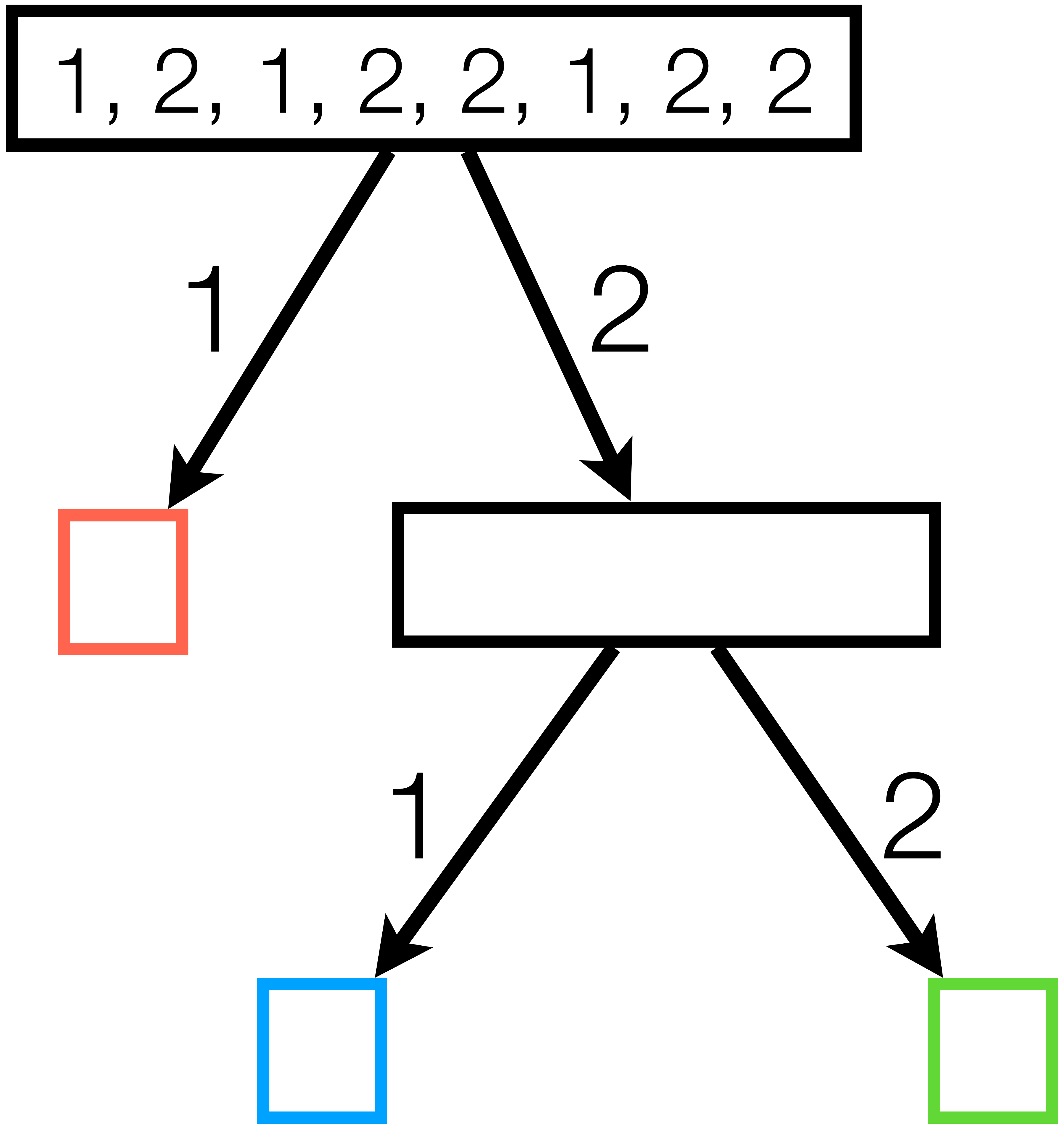

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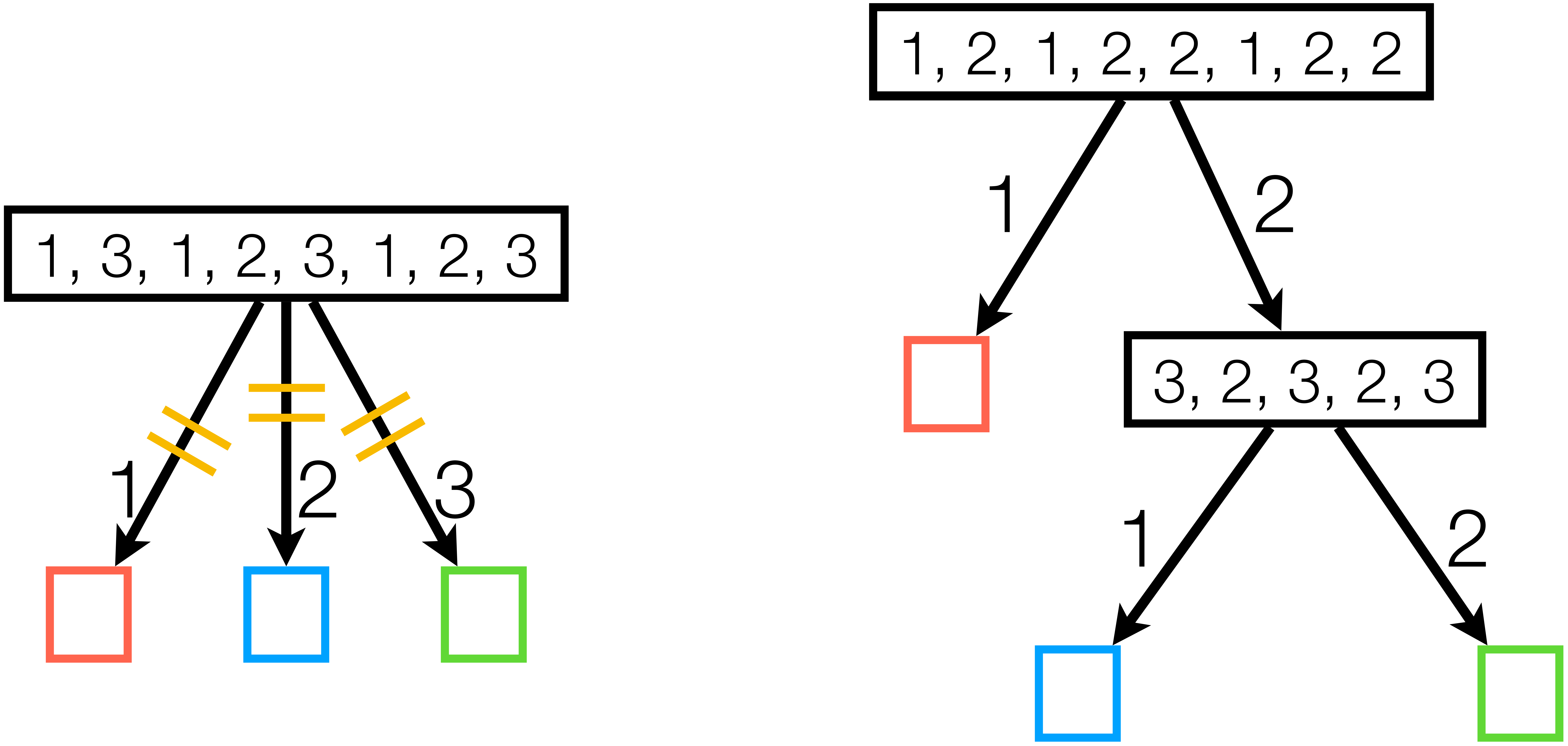

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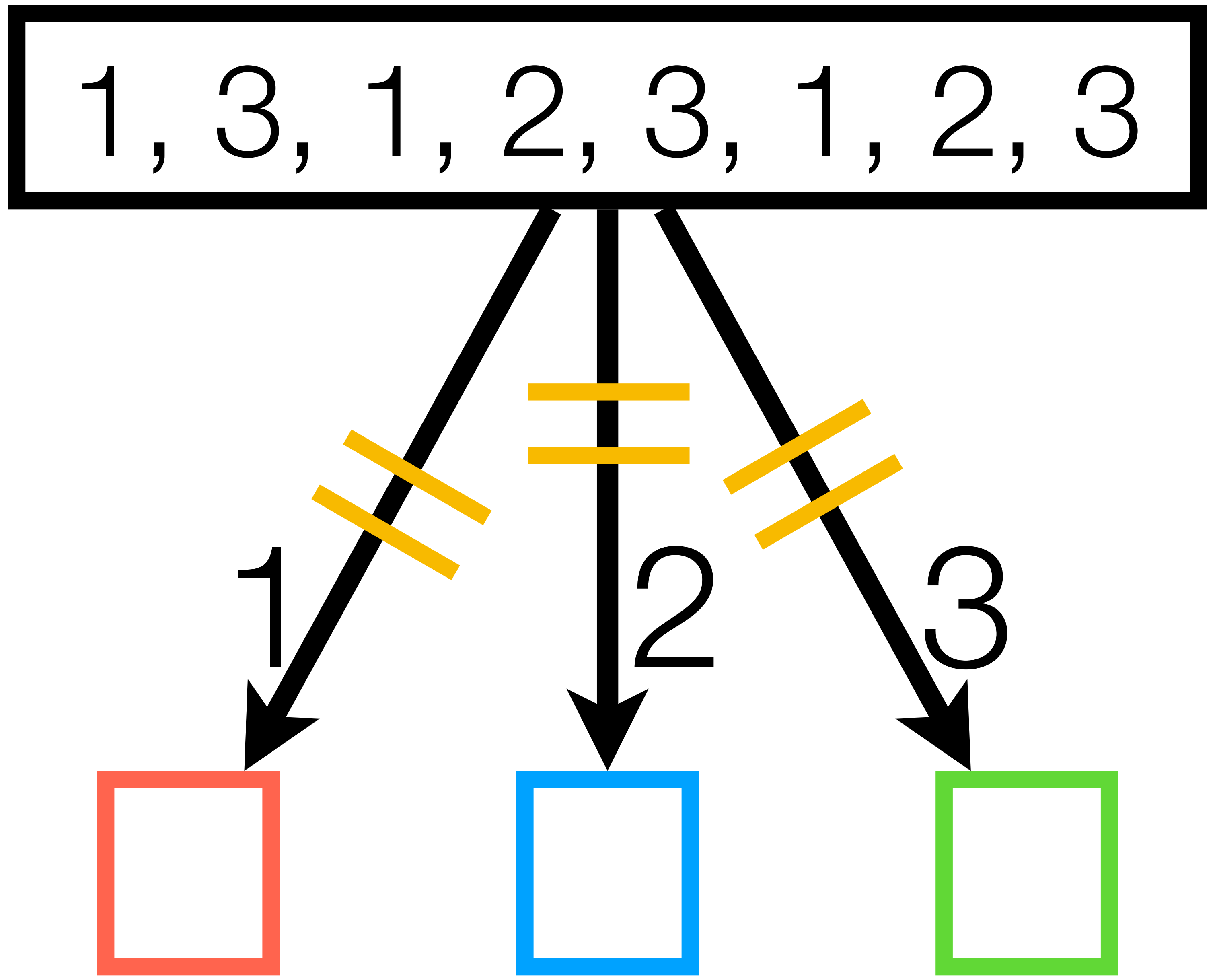

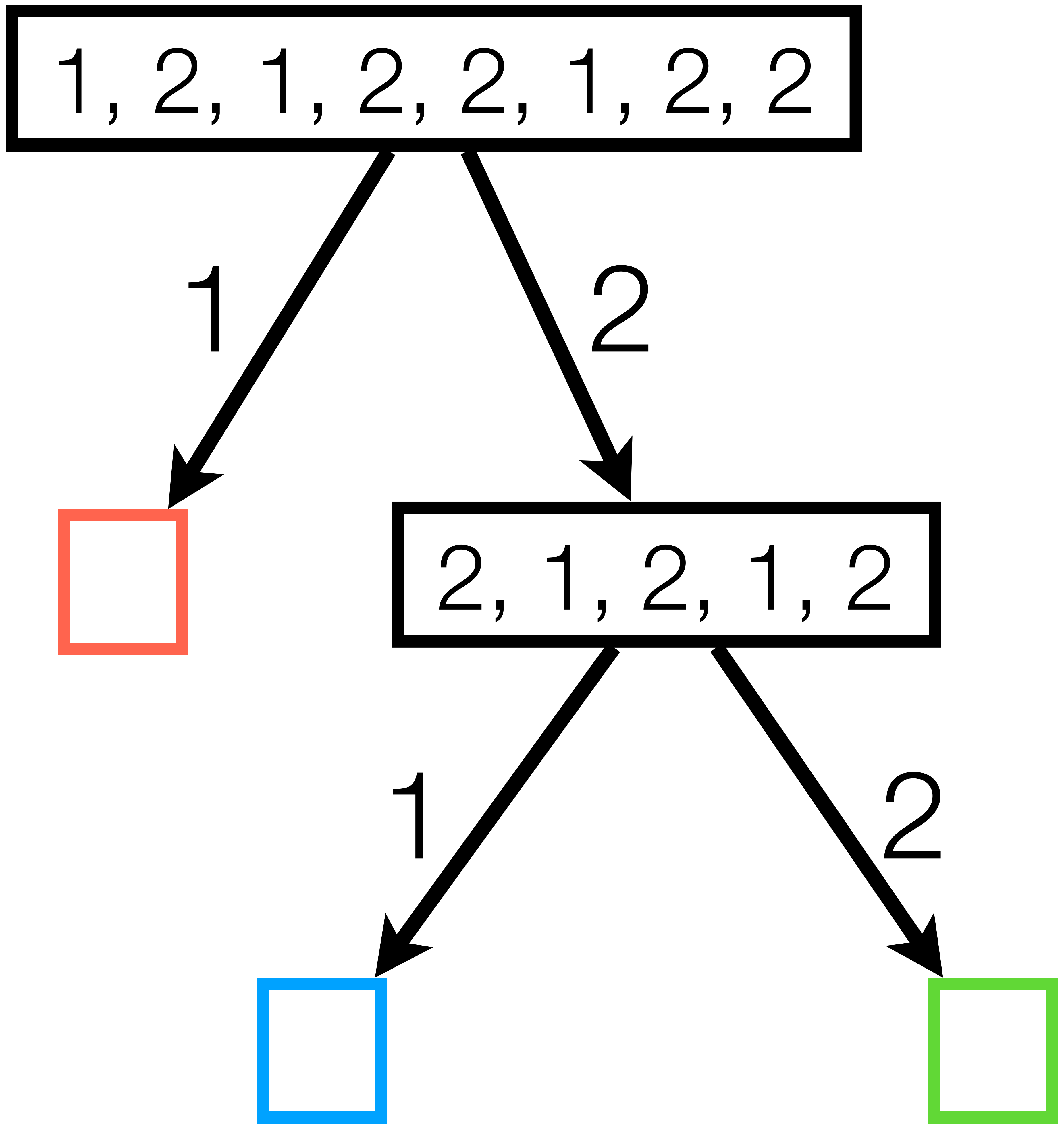

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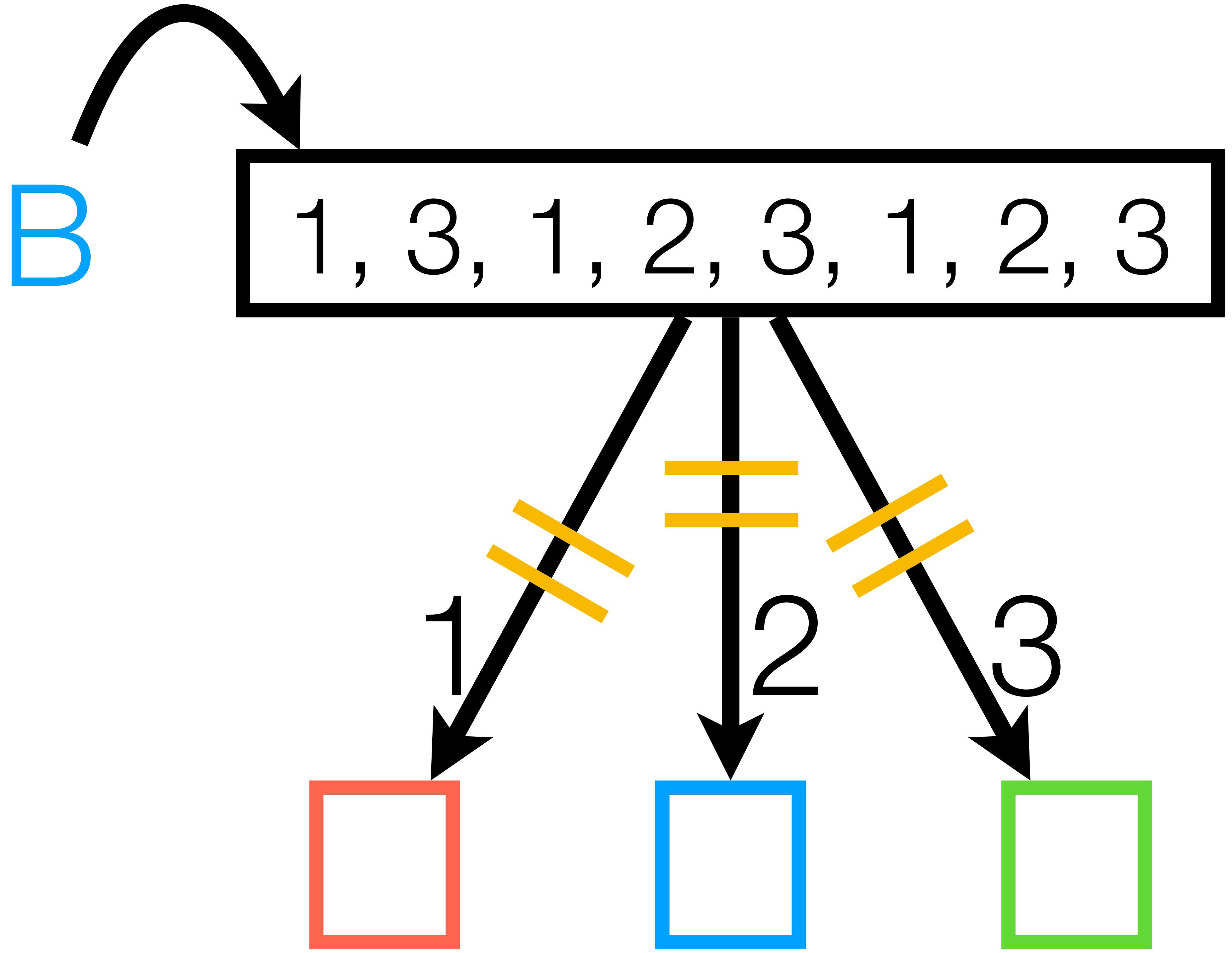
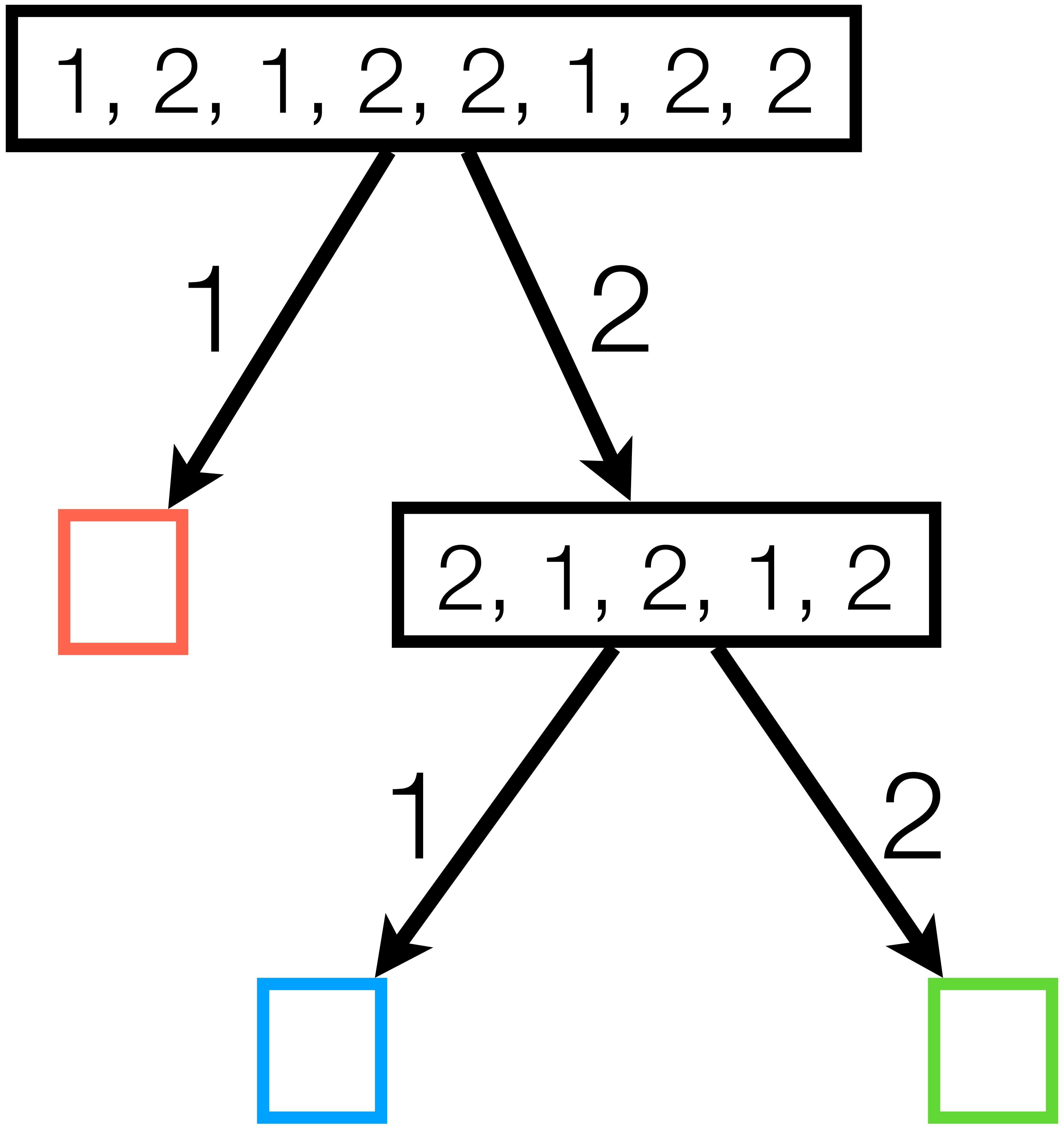
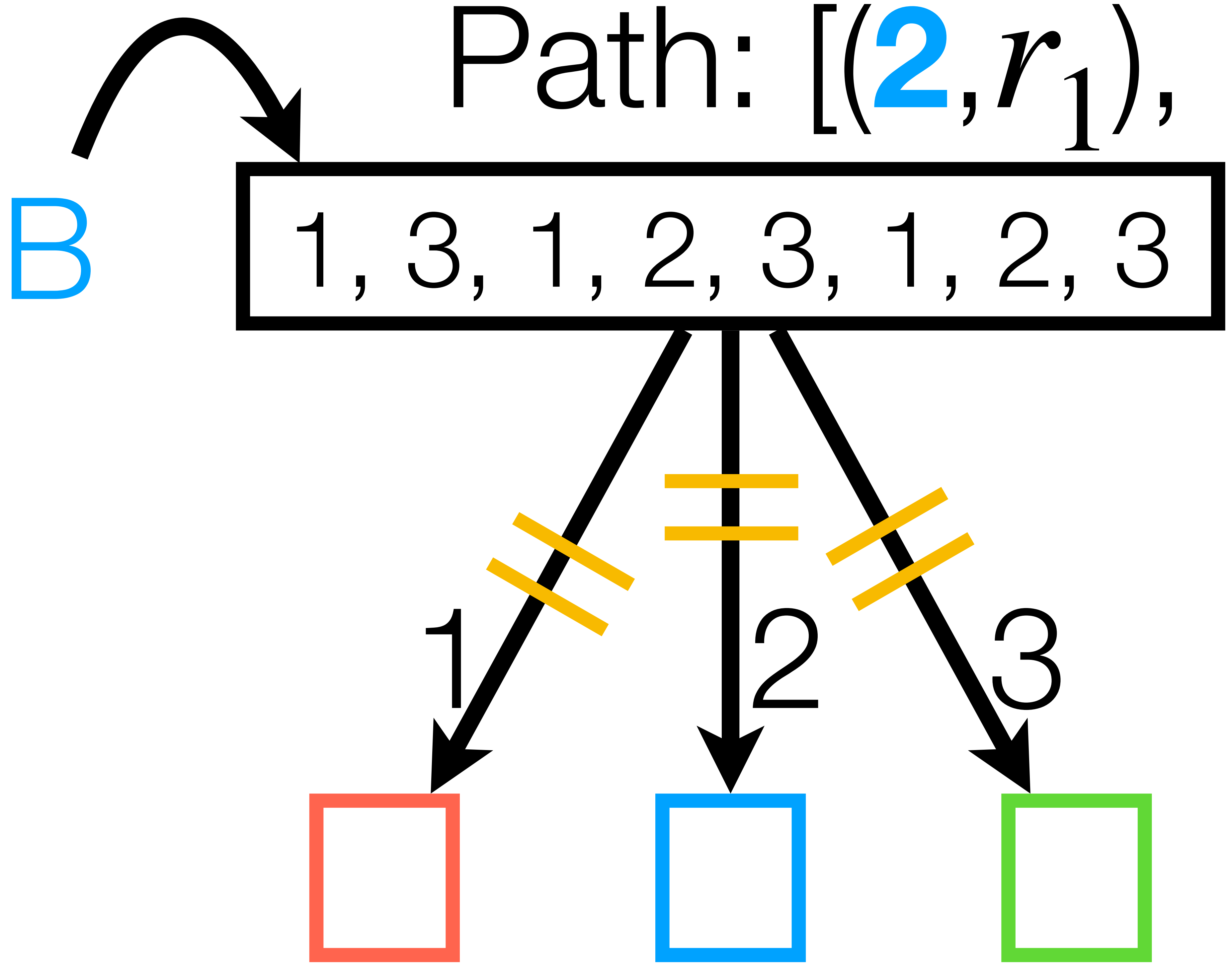
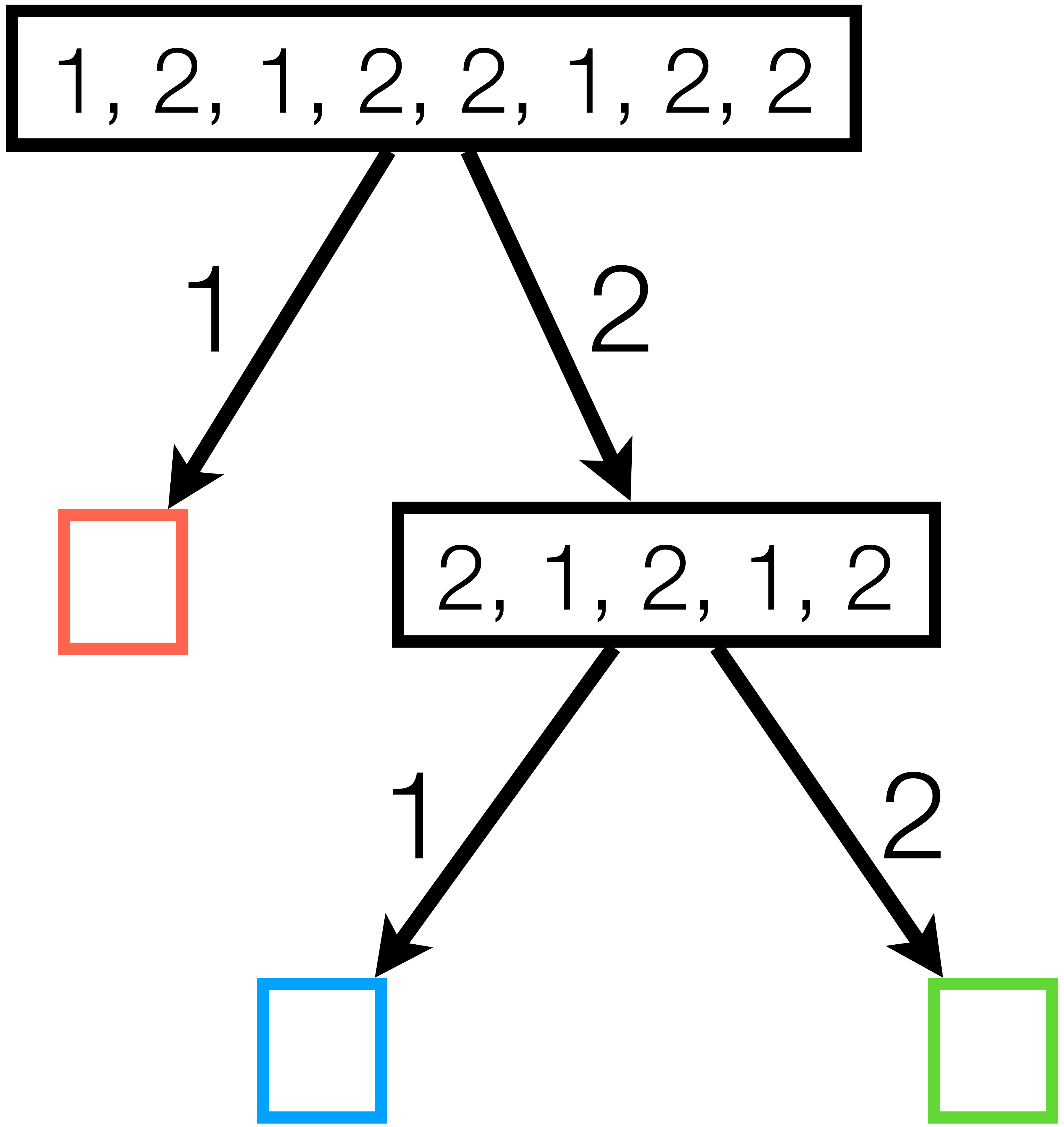




Compiling programs

Path: [($\mathbf{2}$,r_1), …]





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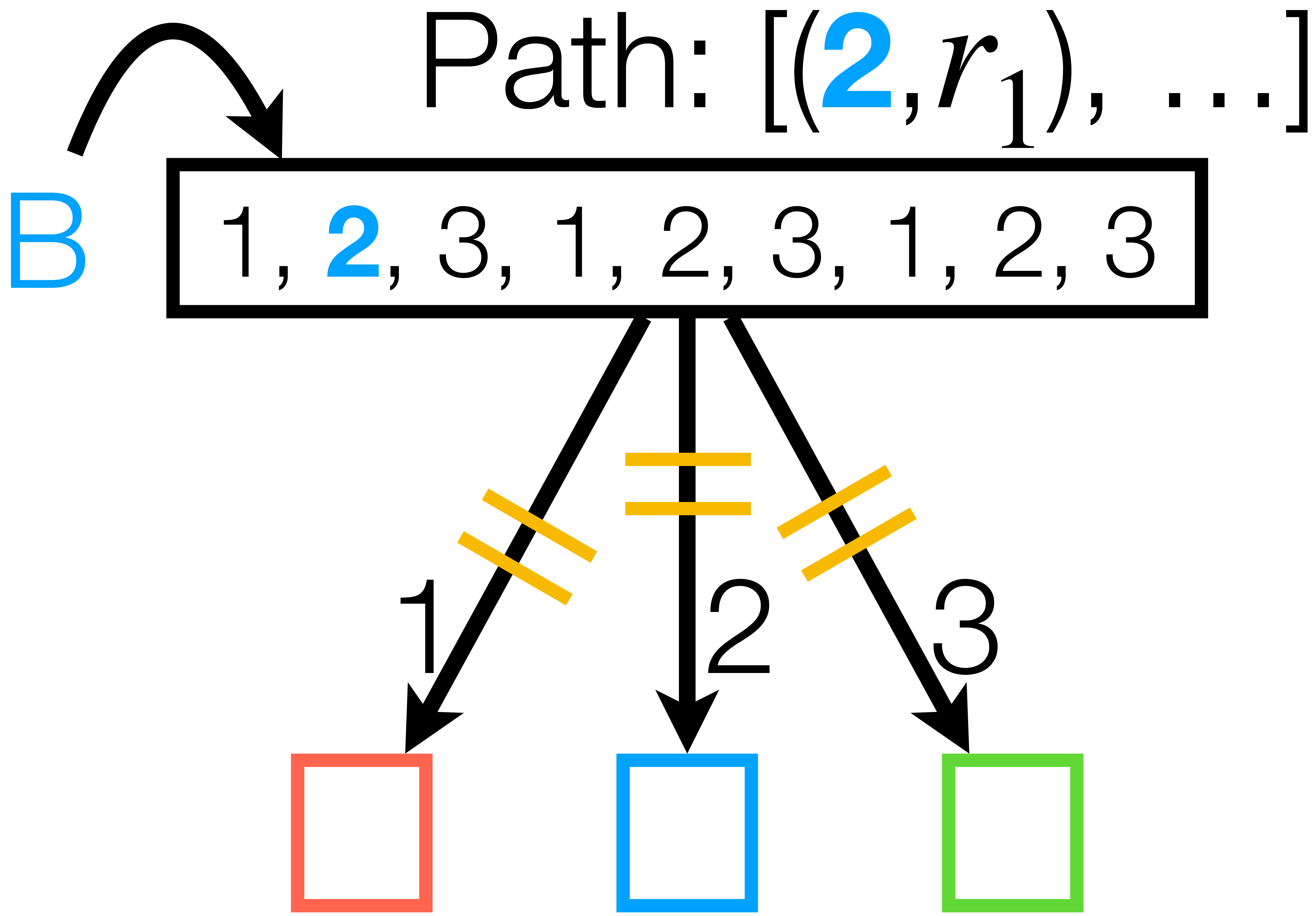

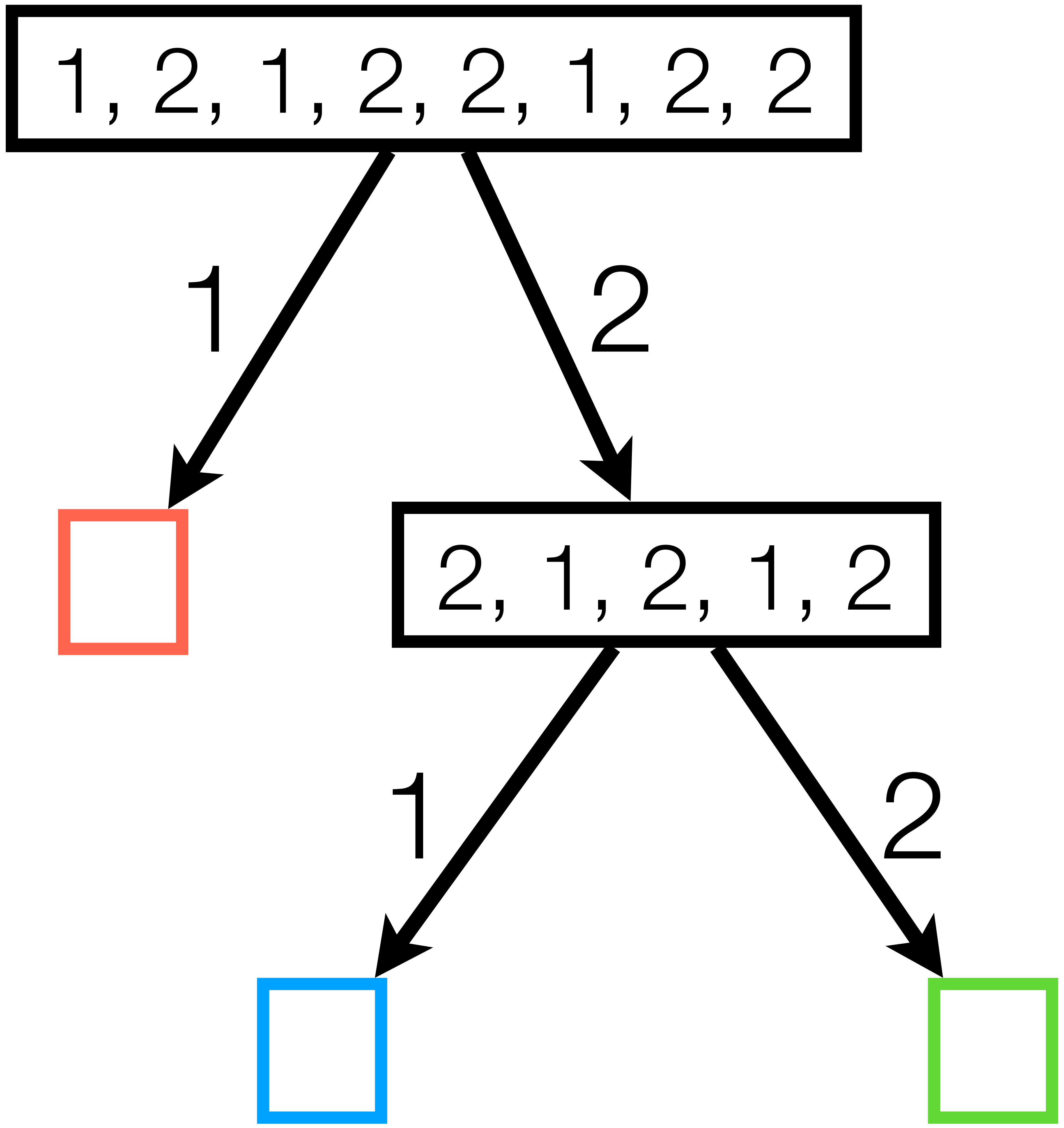

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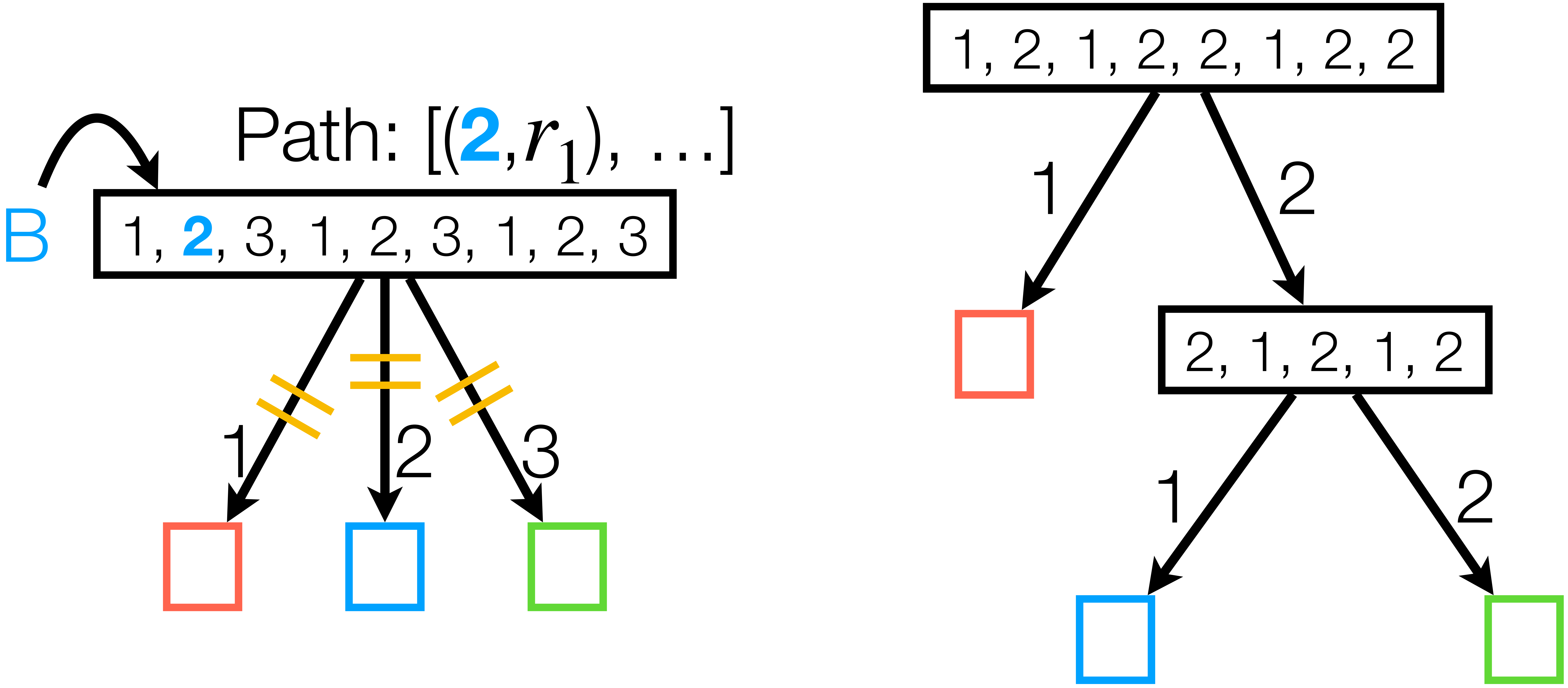

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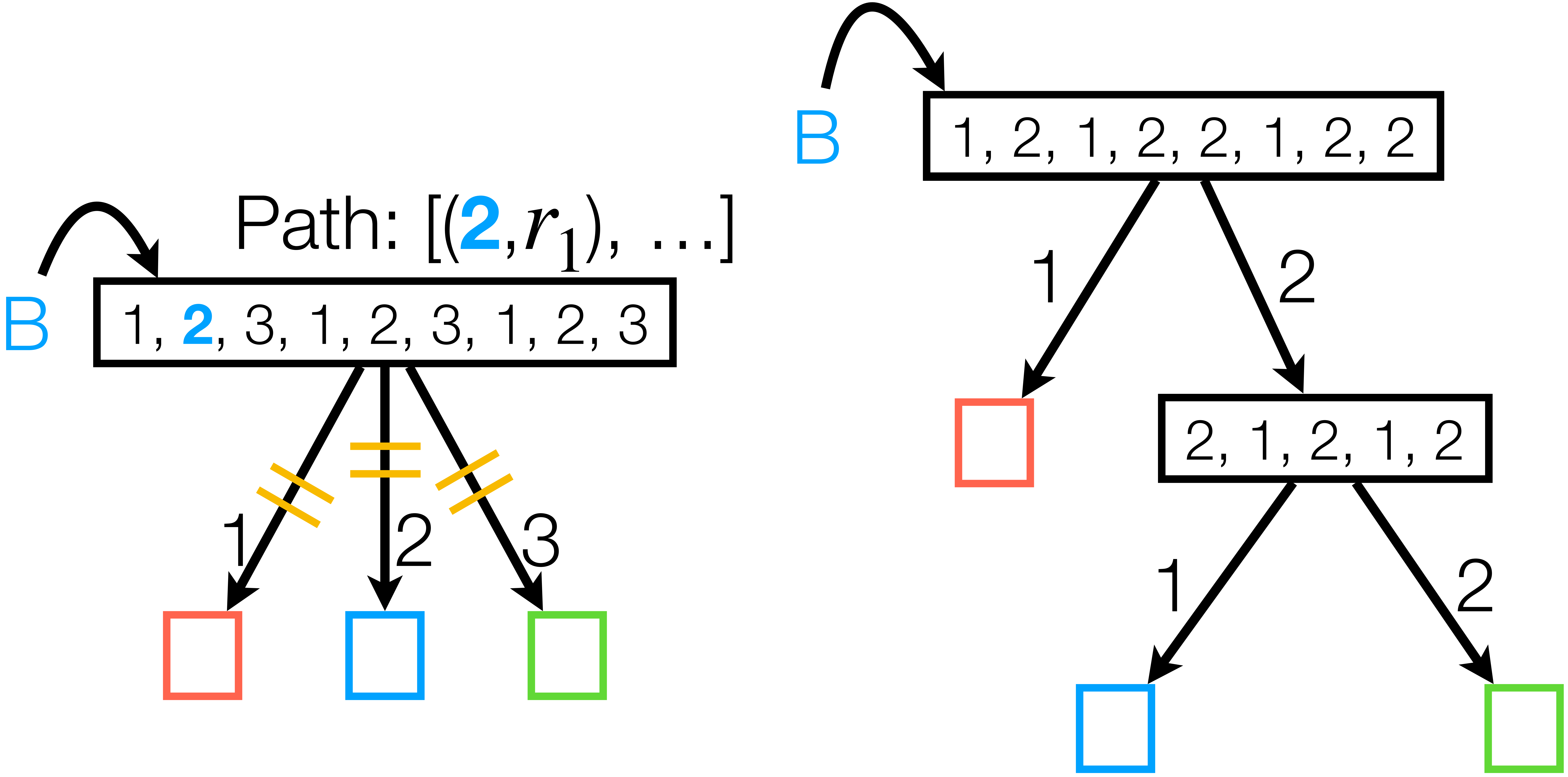

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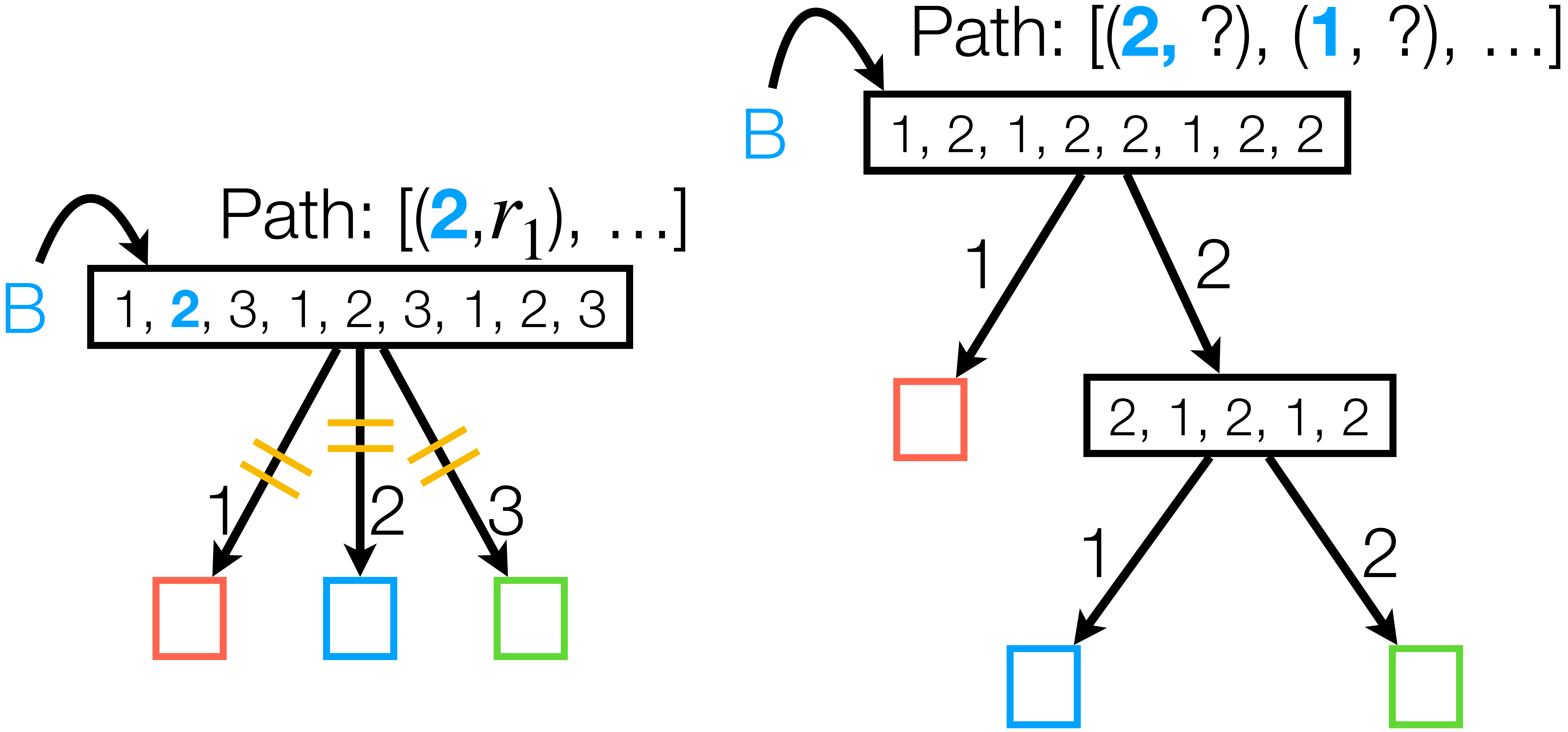

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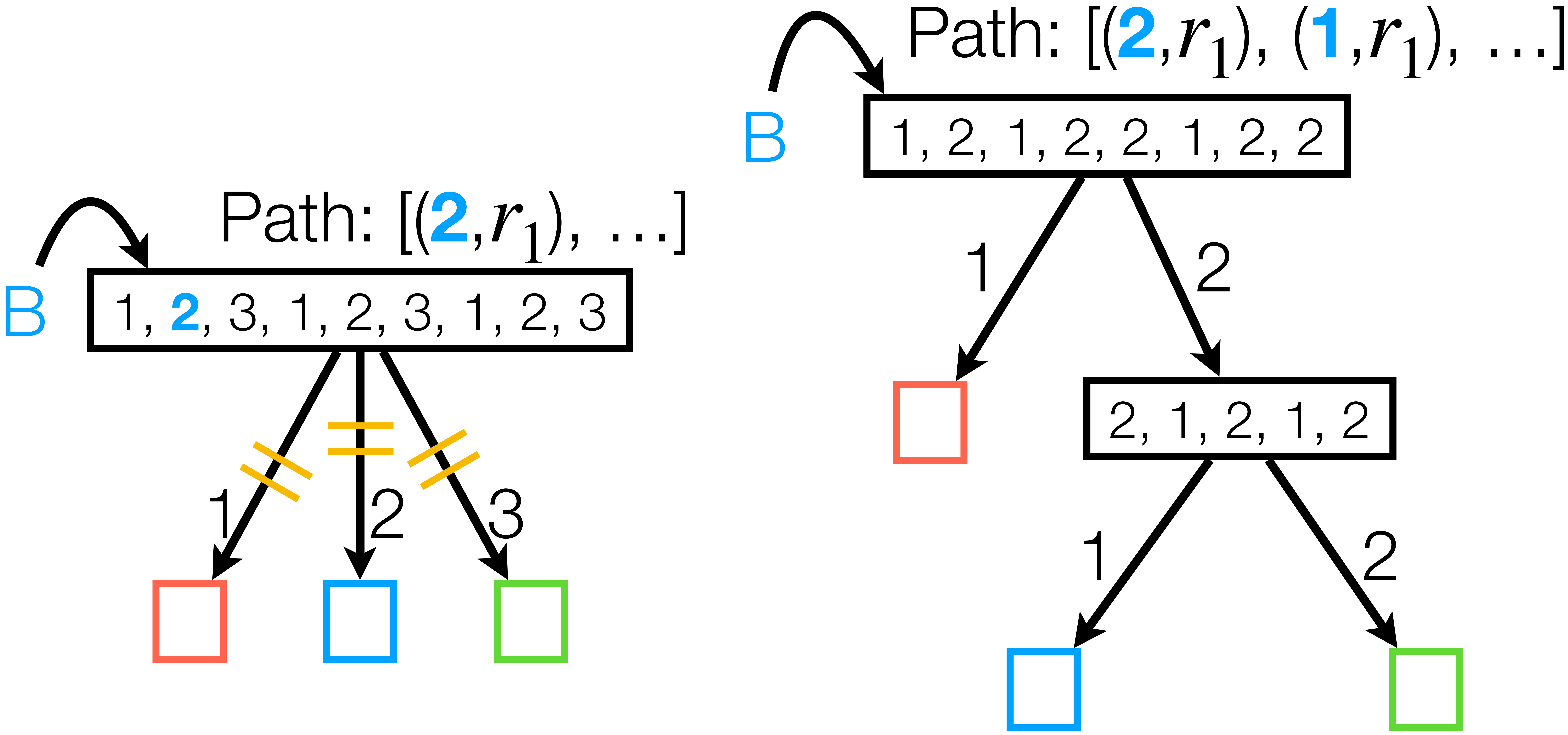

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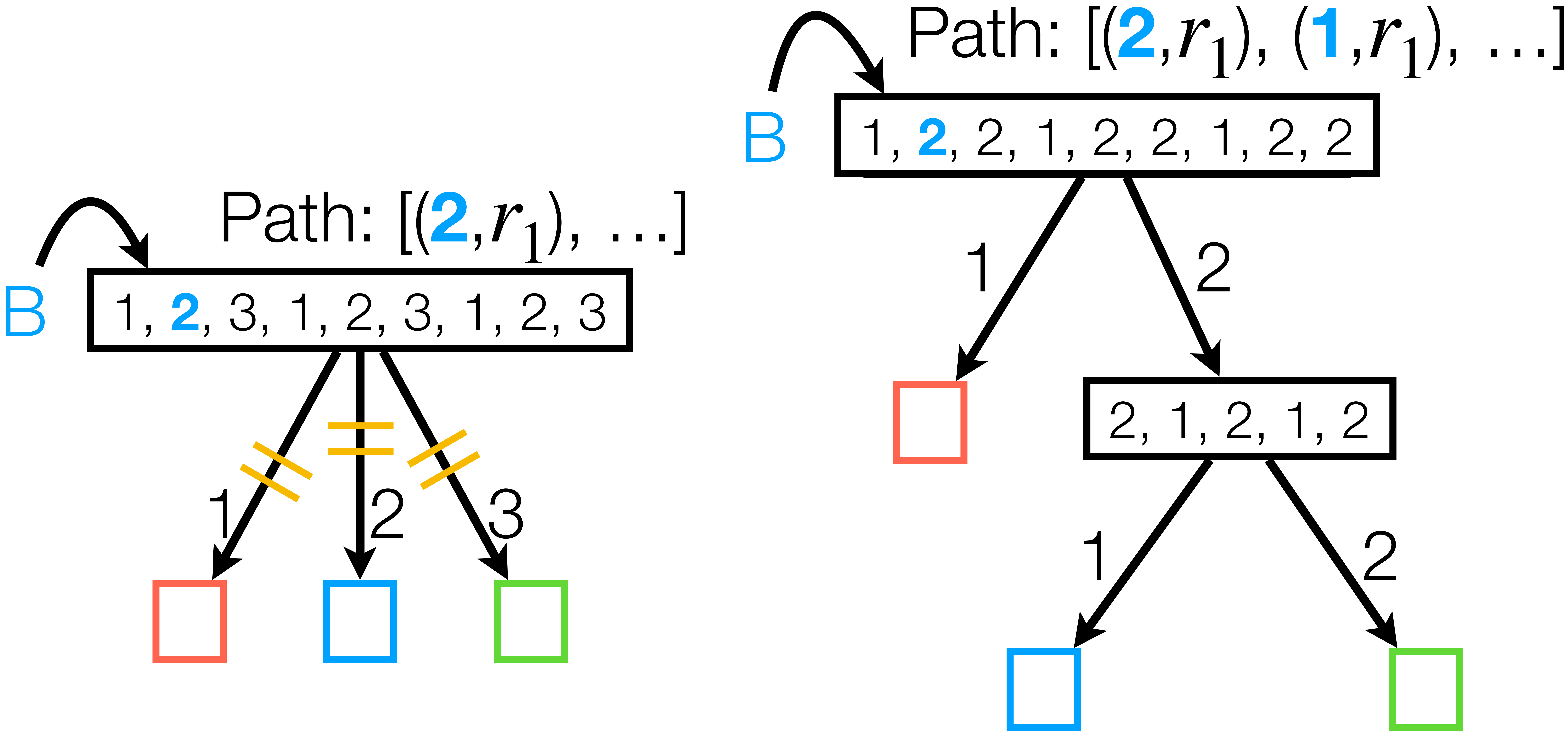

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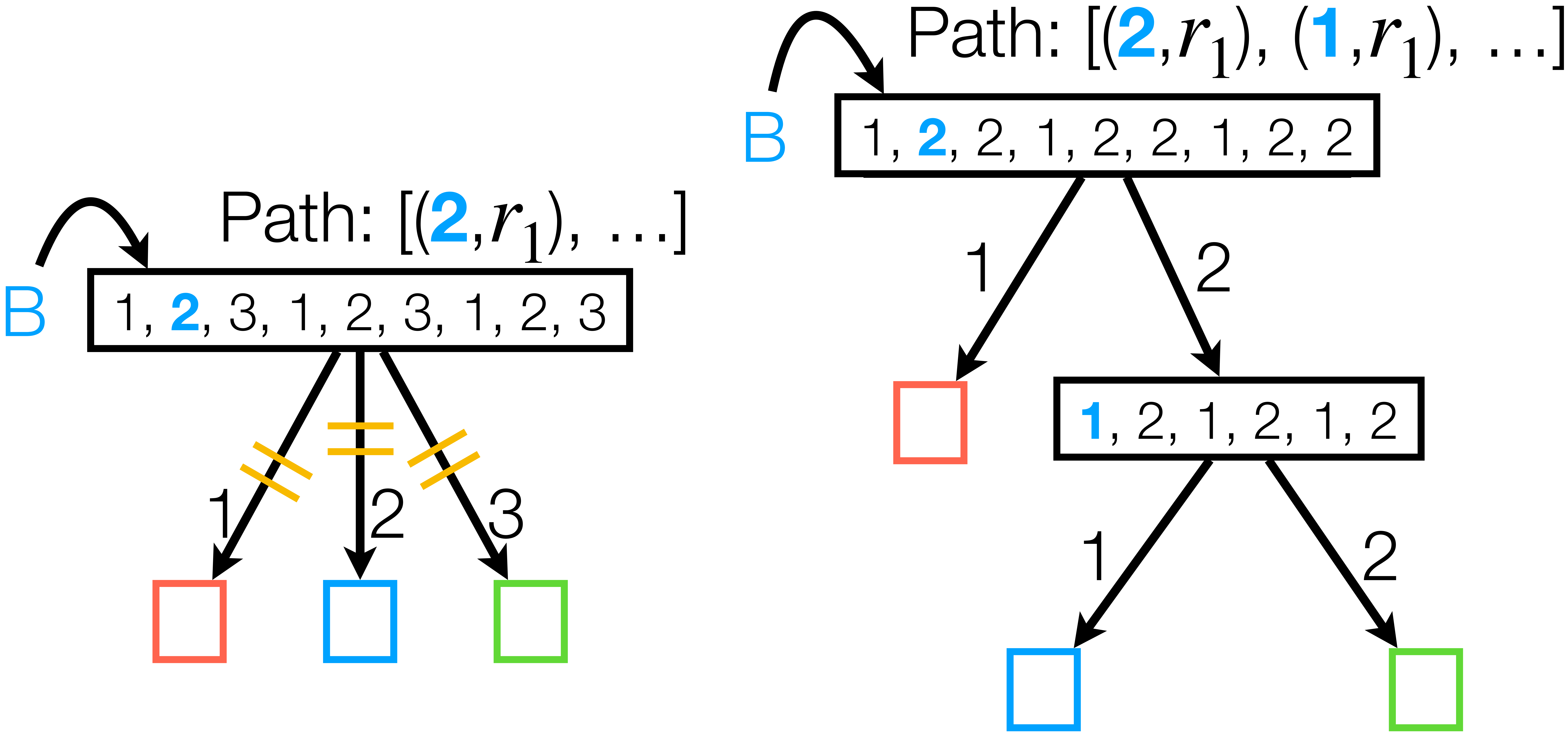

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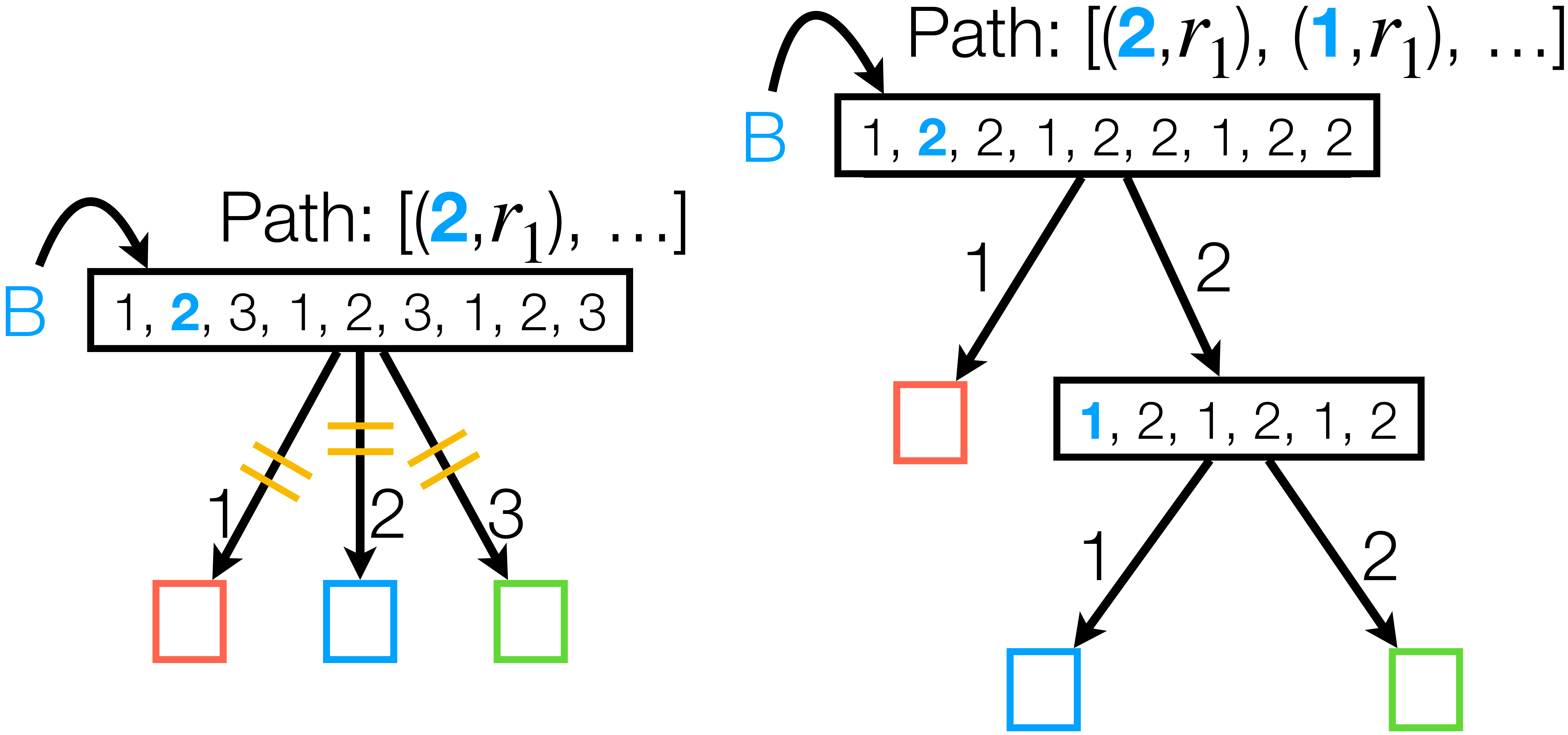
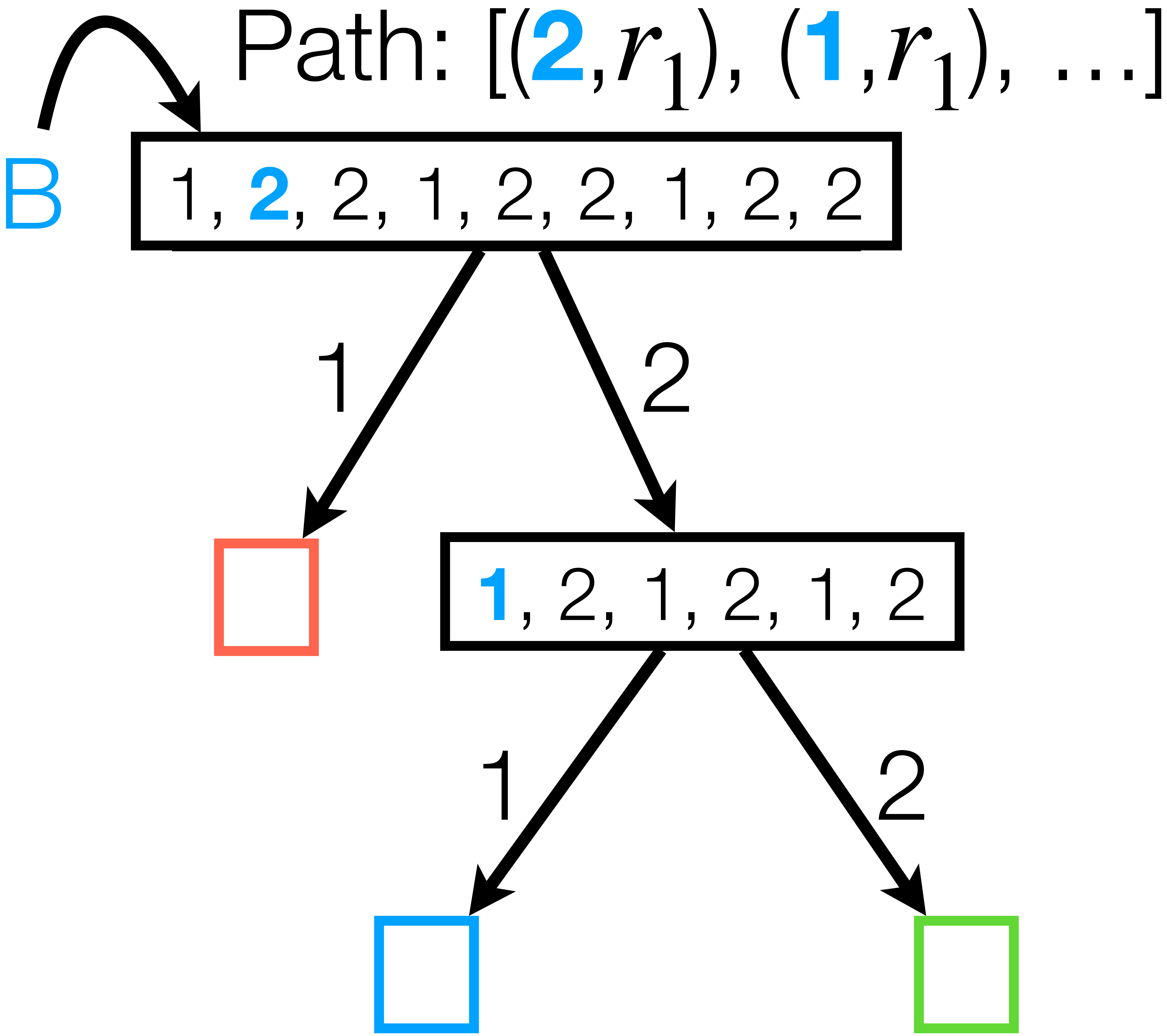
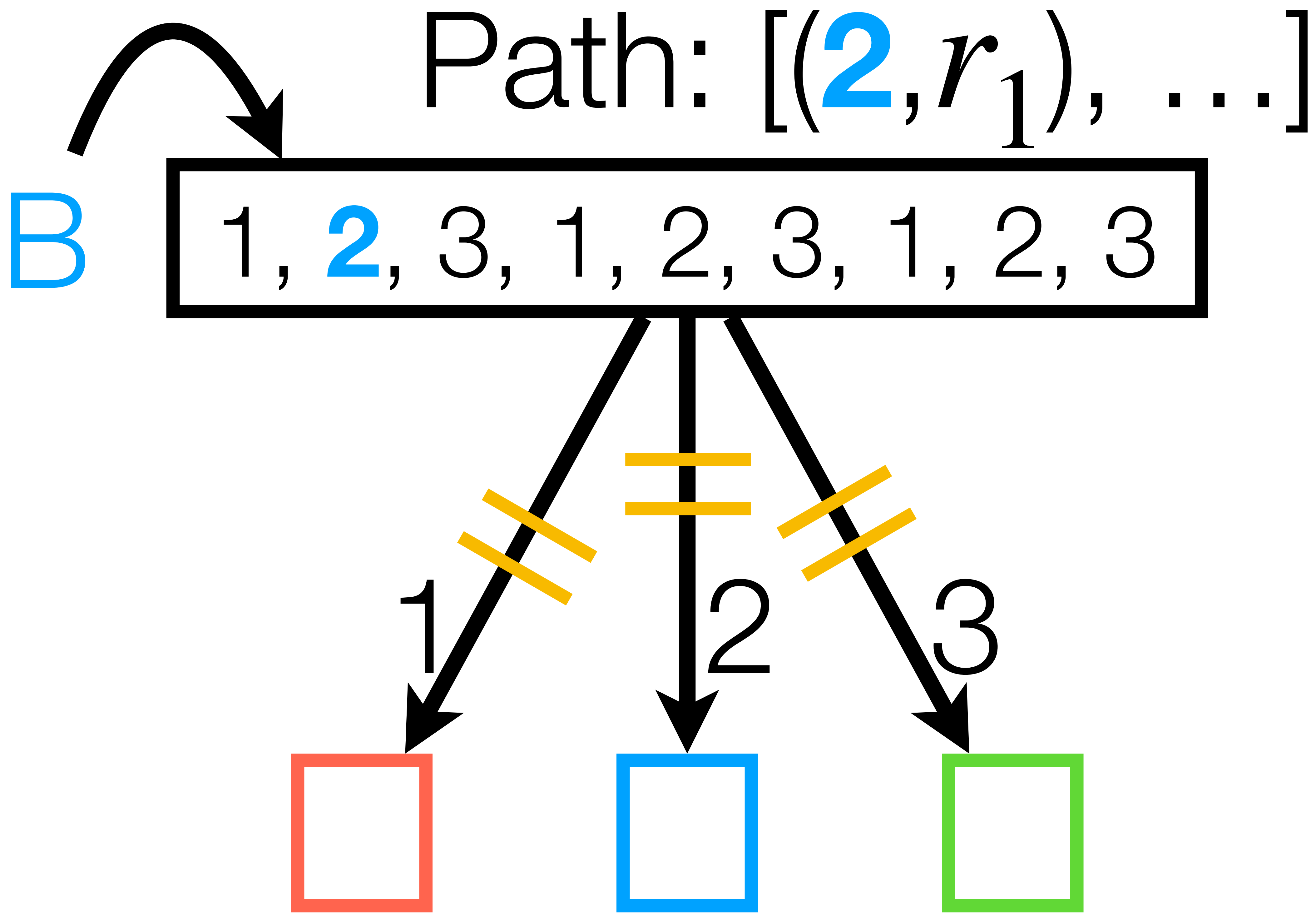


Given an embedding, we *lift* it to arrive at a compiler.





Generating embeddings automatically!

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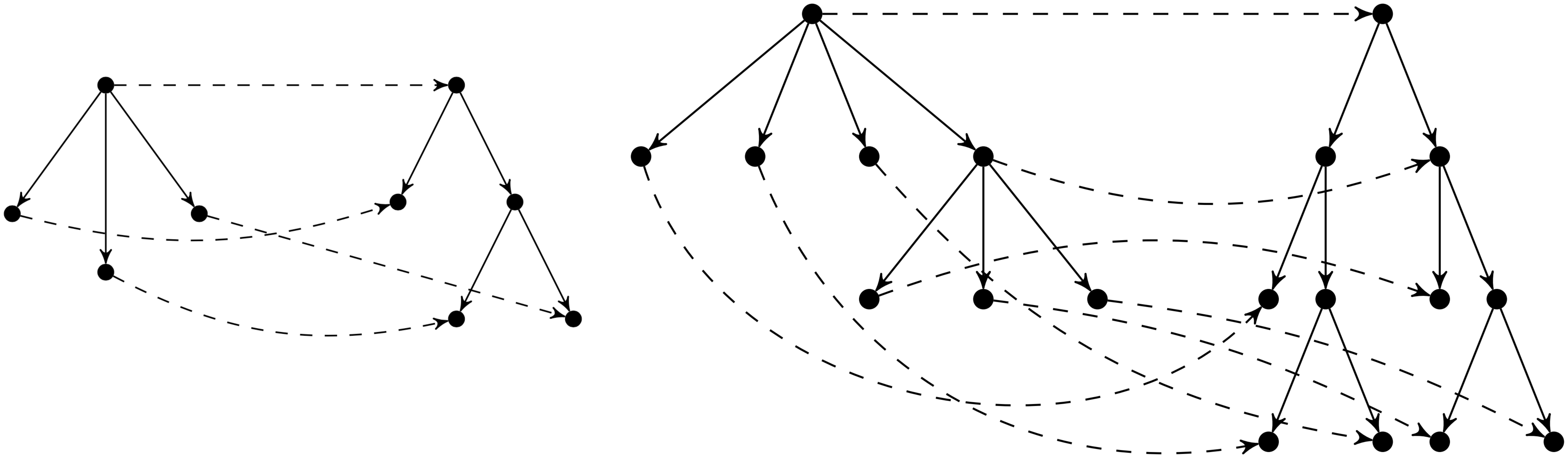
Homomorphic embedding.
Map root to root, leaves to leaves. Respect ancestry.



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Two new algorithms,
both starting with heterogeneous source trees.

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1. If target tree is regular d-ary for some d.

Generating embeddings automatically!

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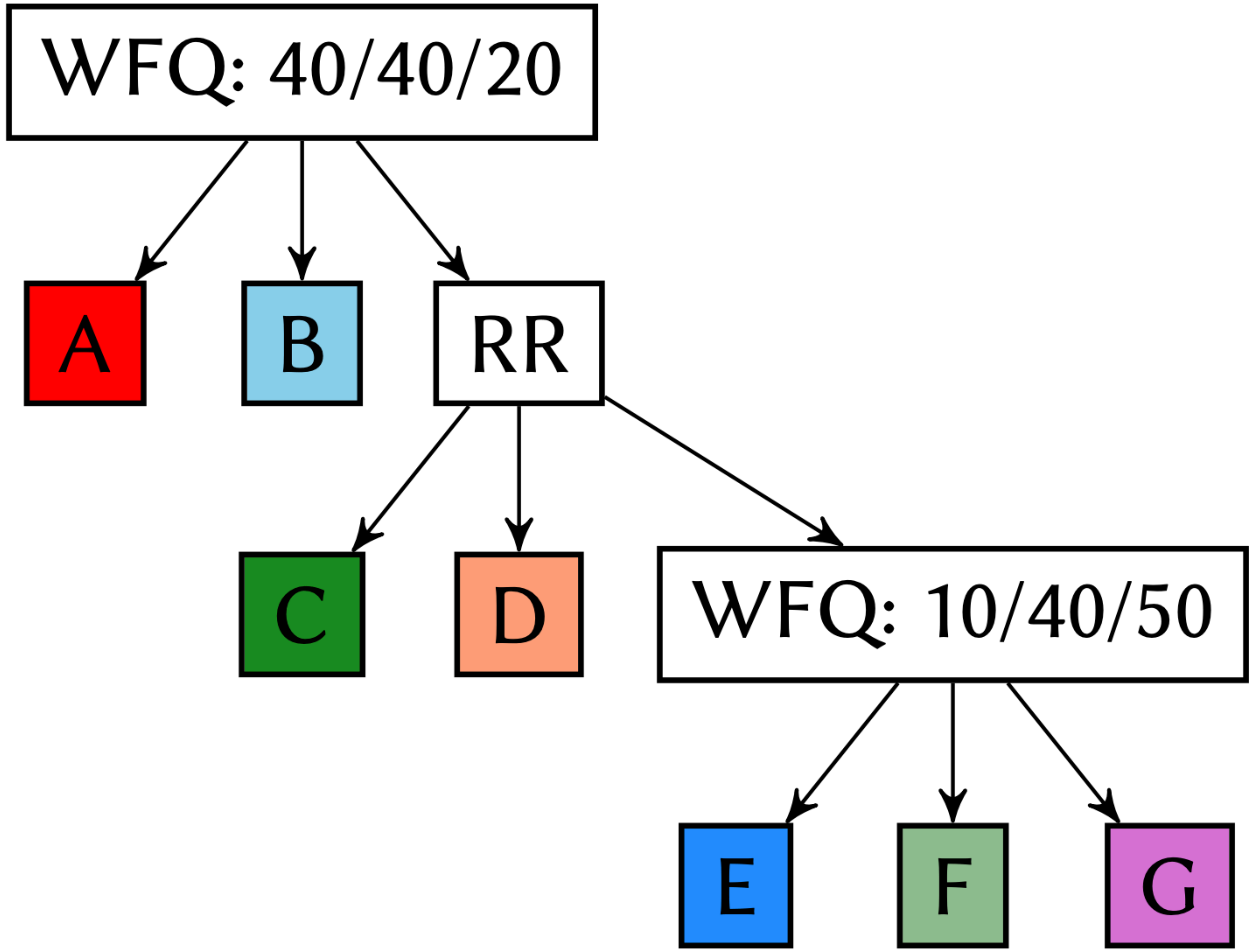
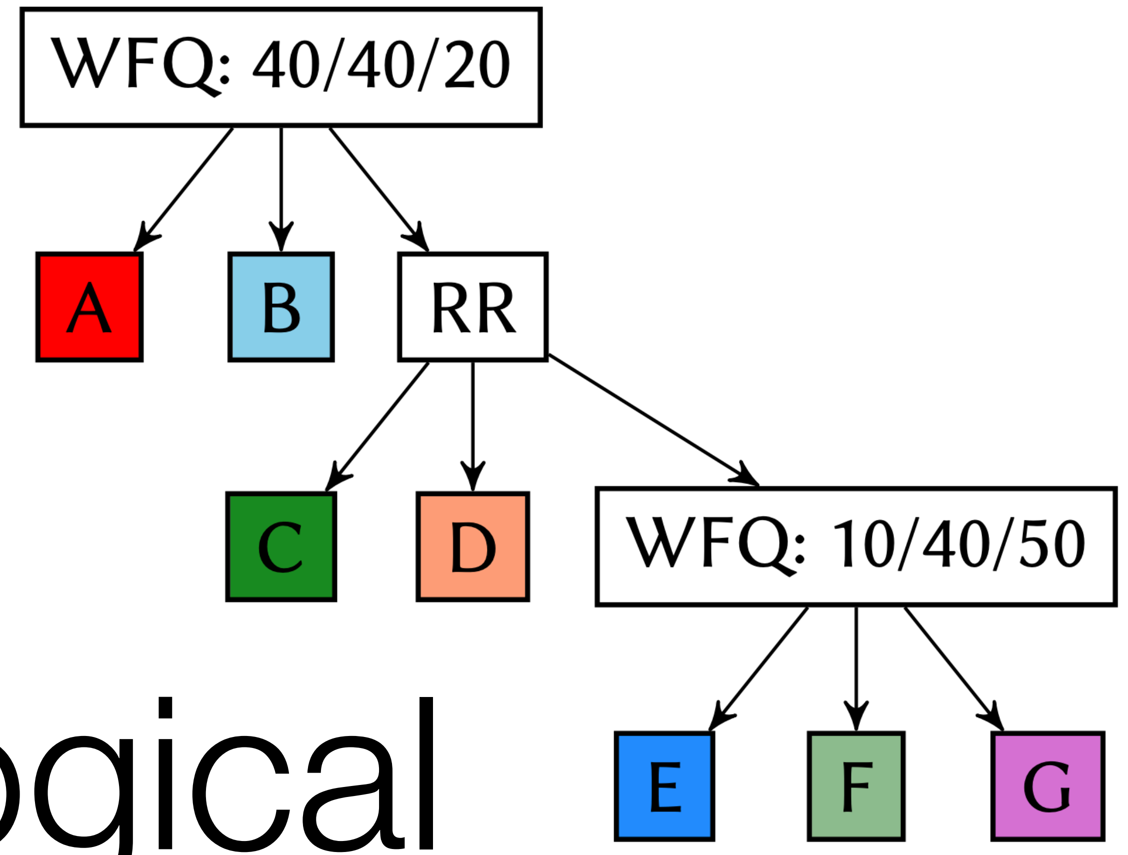
Two new algorithms,
both starting with heterogeneous source trees.

1. If target tree is regular *d*-ary for some *d.*
2. If target tree is itself heterogeneous.

Workflow

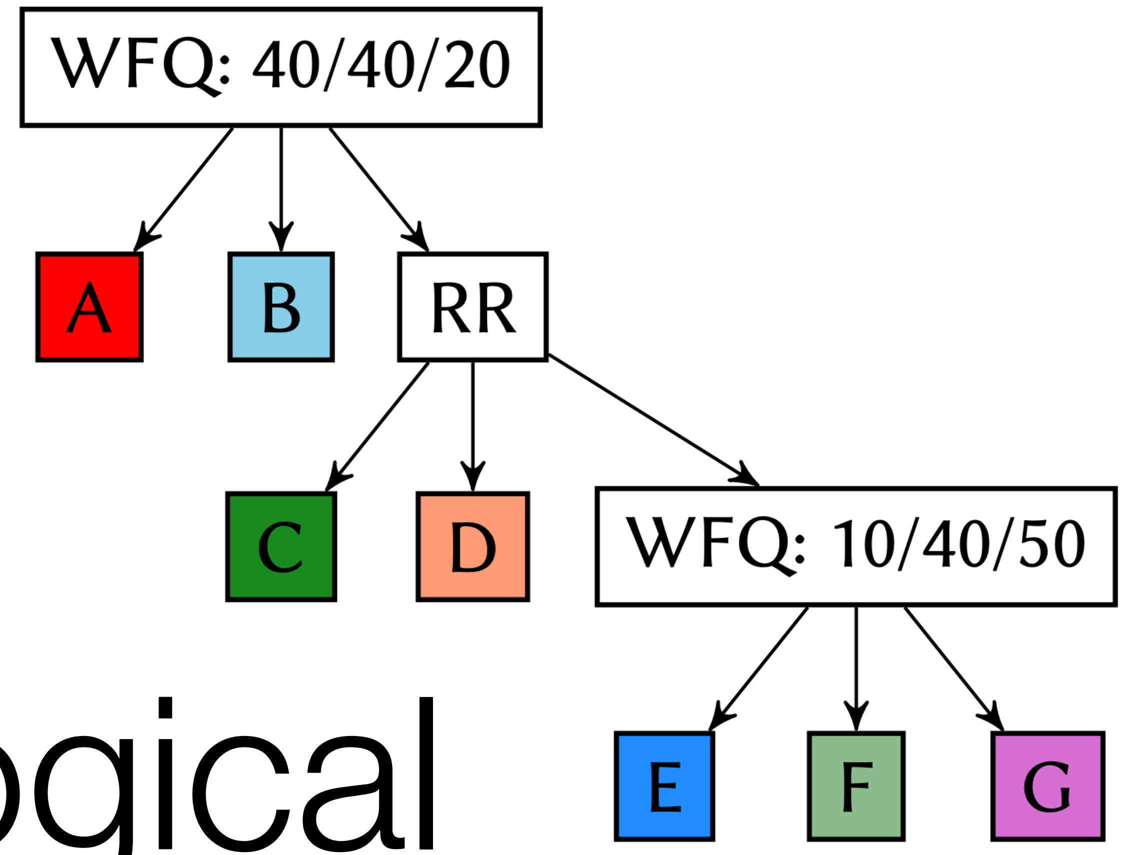


Workflow



logical

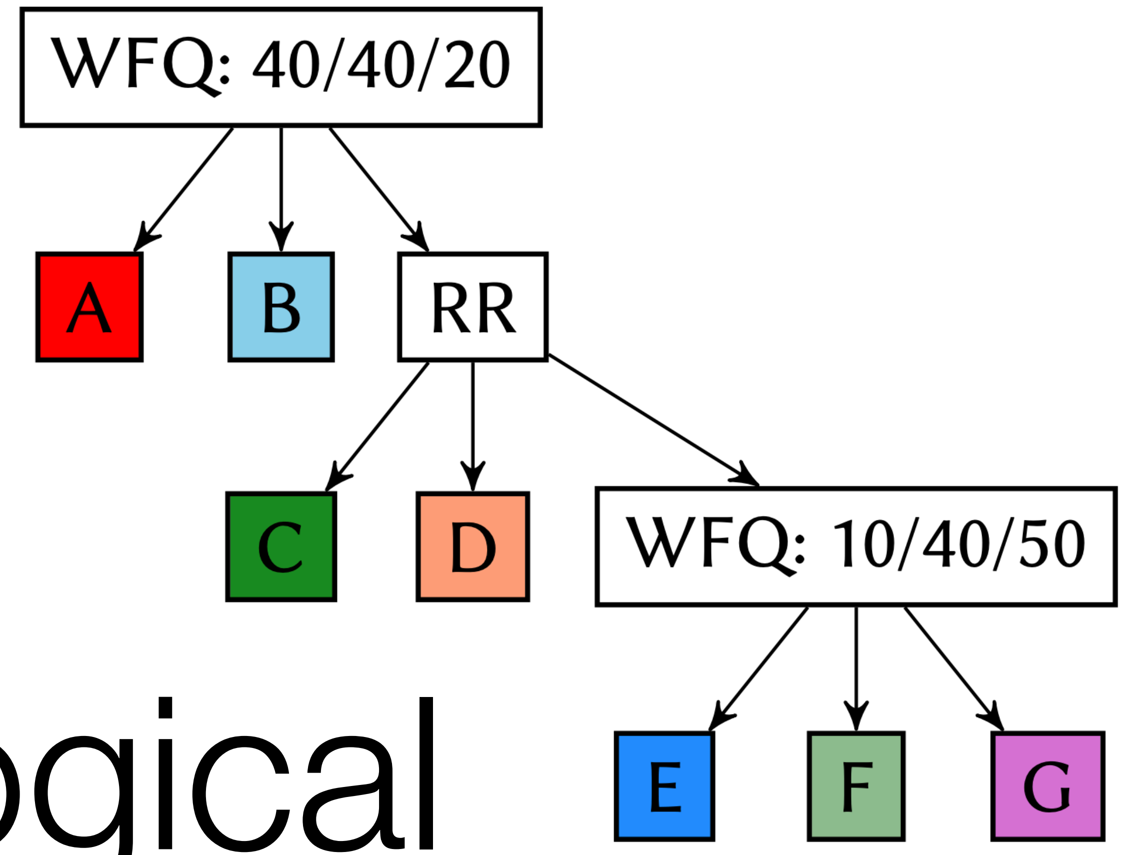
Workflow



logical

But the hardware supports
a regular-branching binary tree.

Workflow



logical

But the hardware supports
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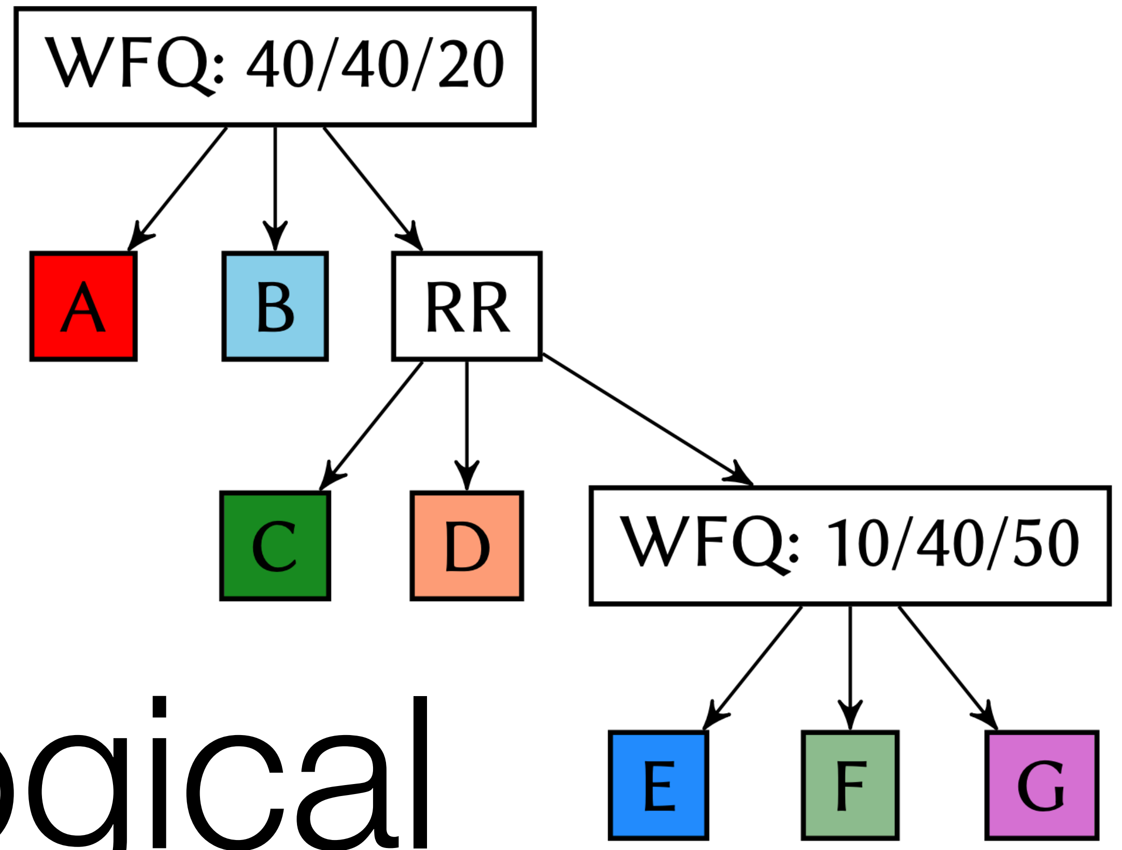
Here’s how I’ll use that tree.

Workflow



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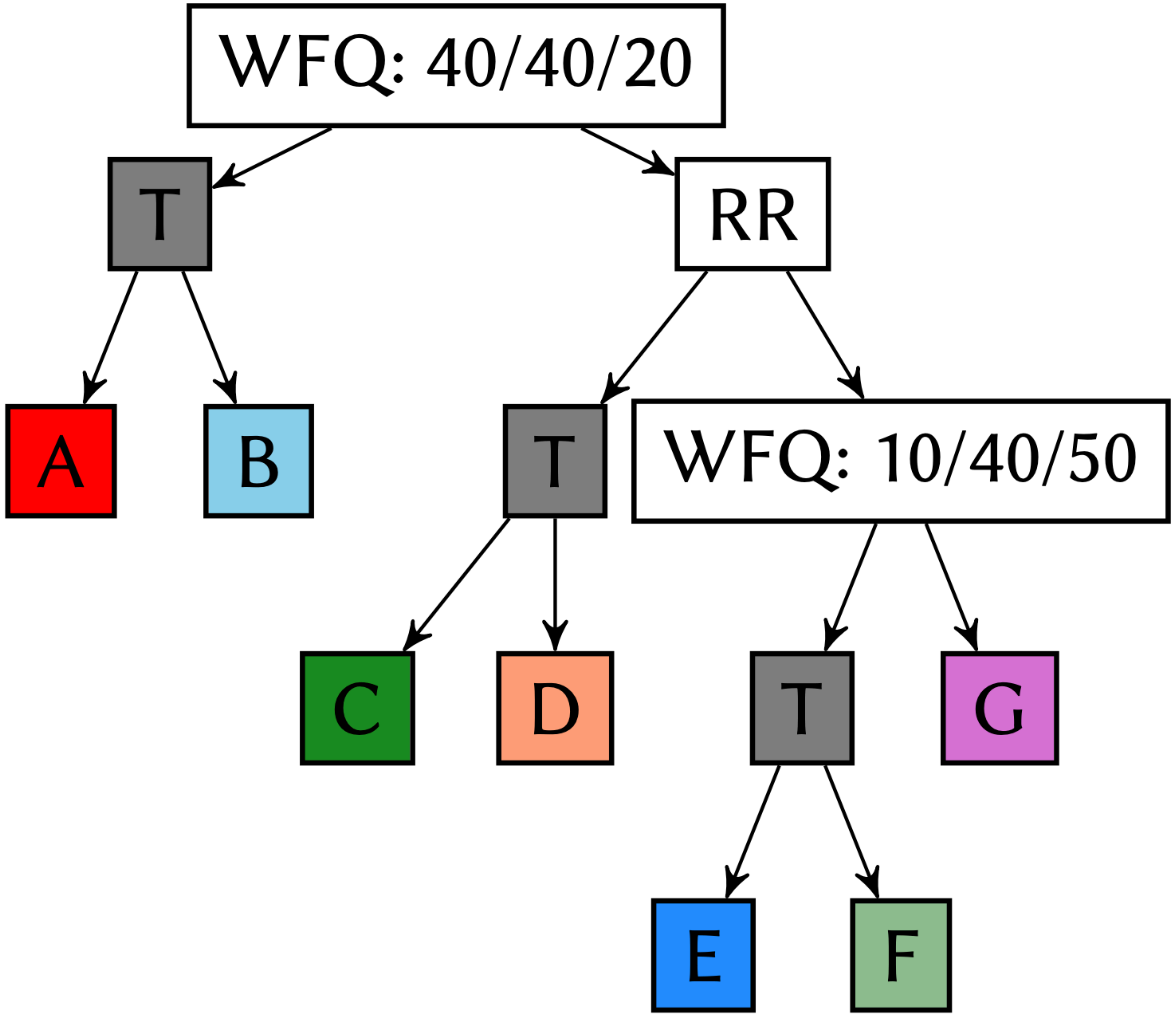

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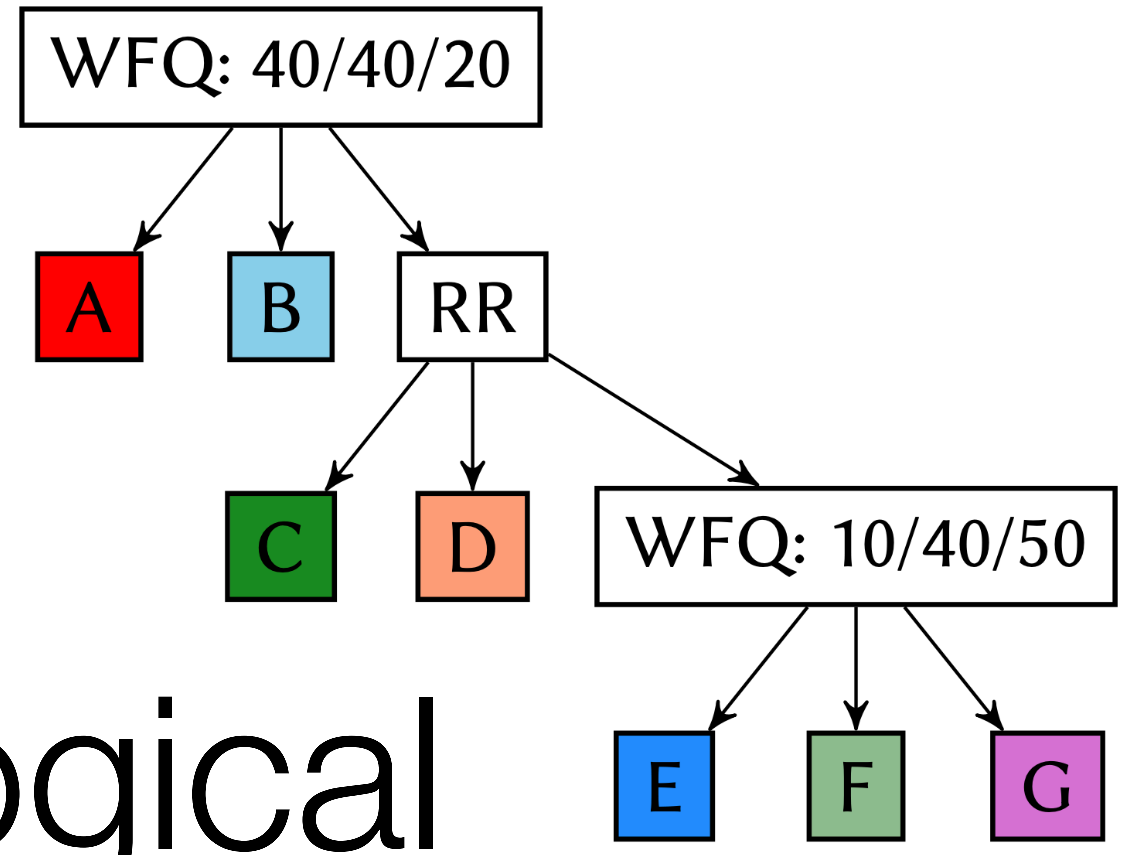

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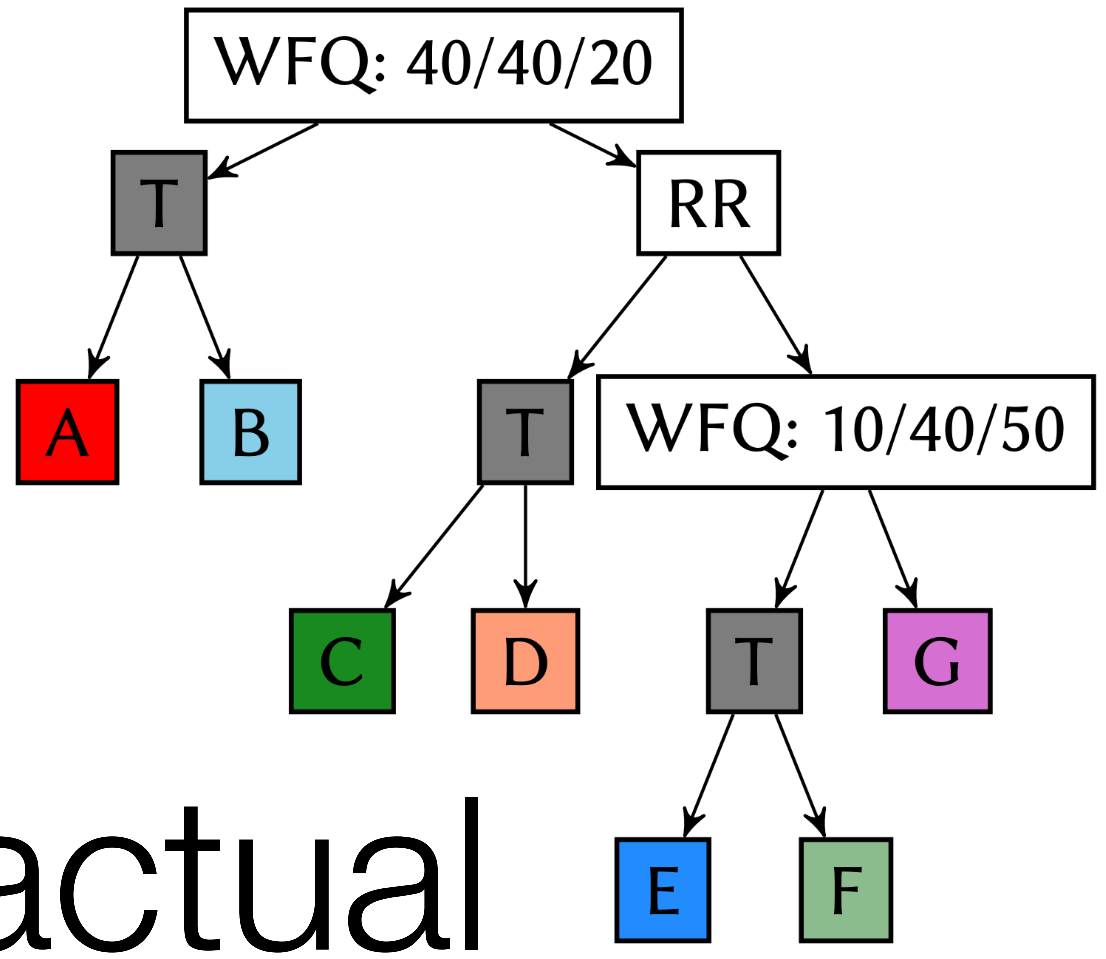
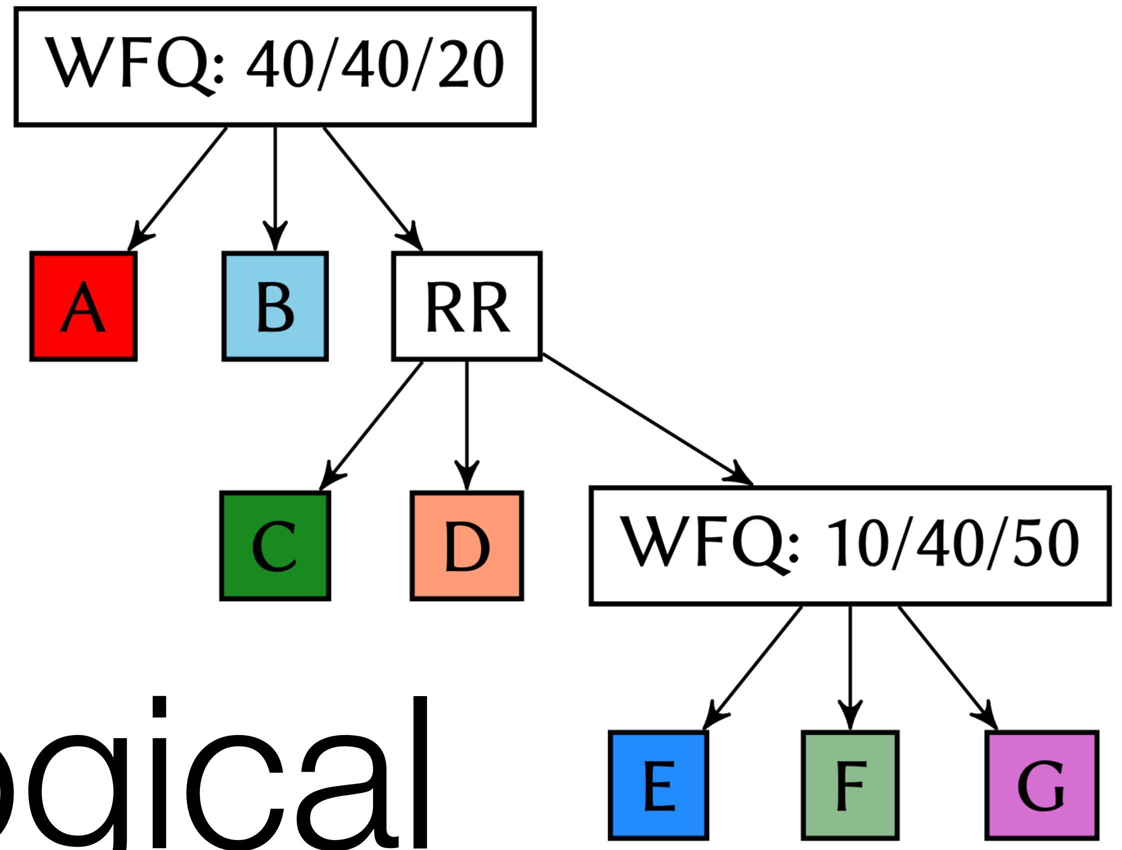


actual

Simulation



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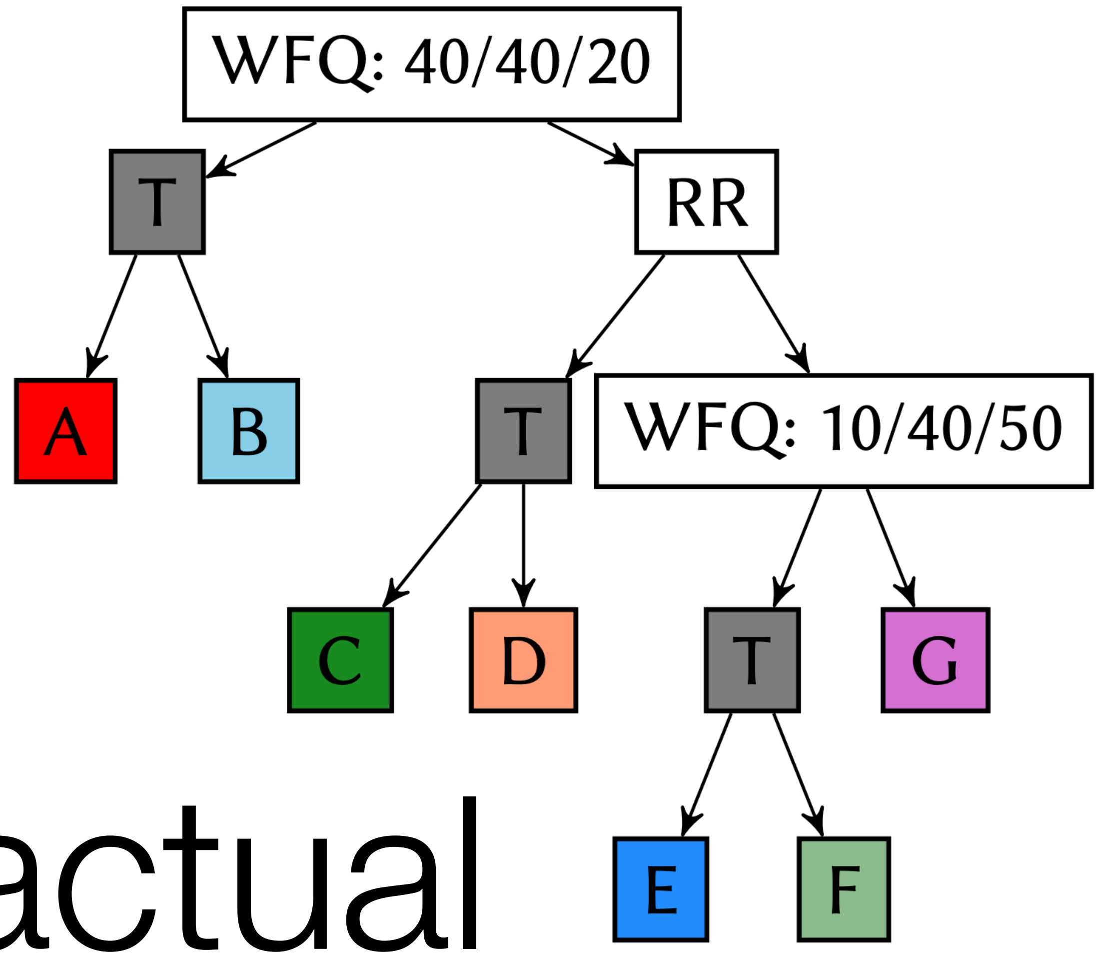

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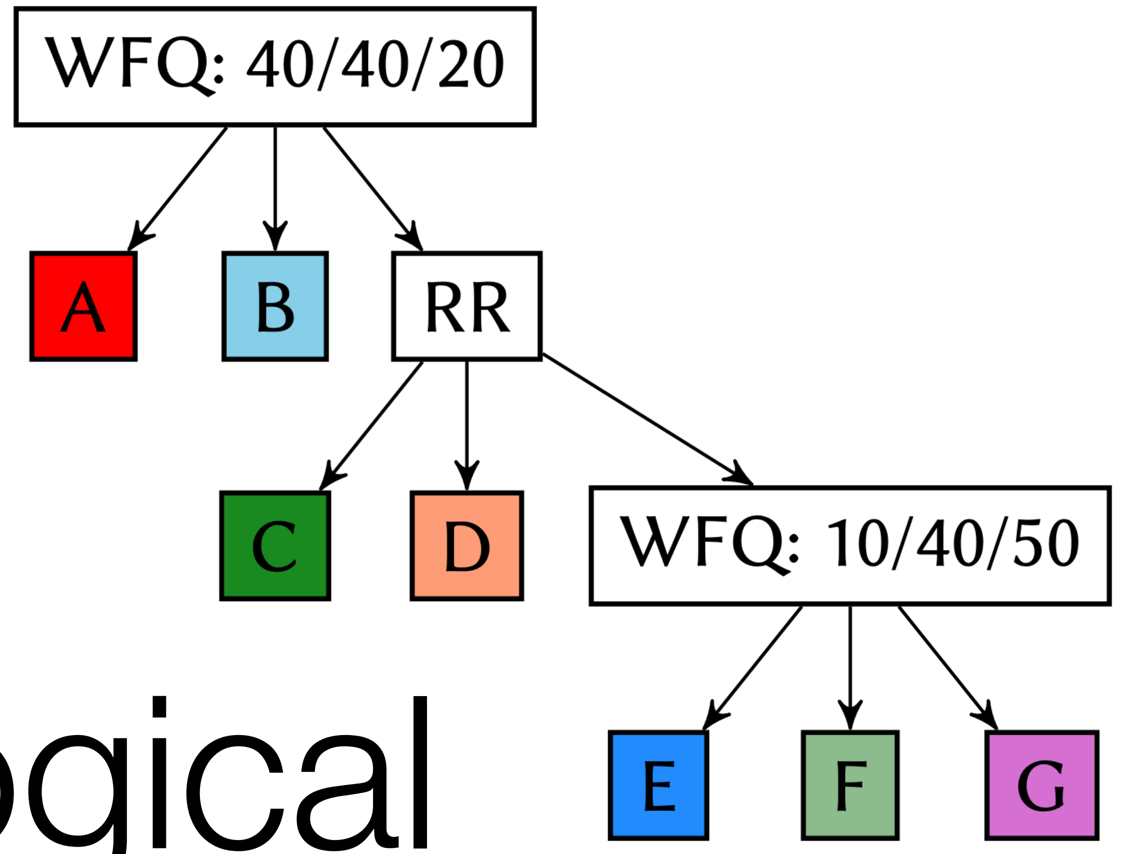

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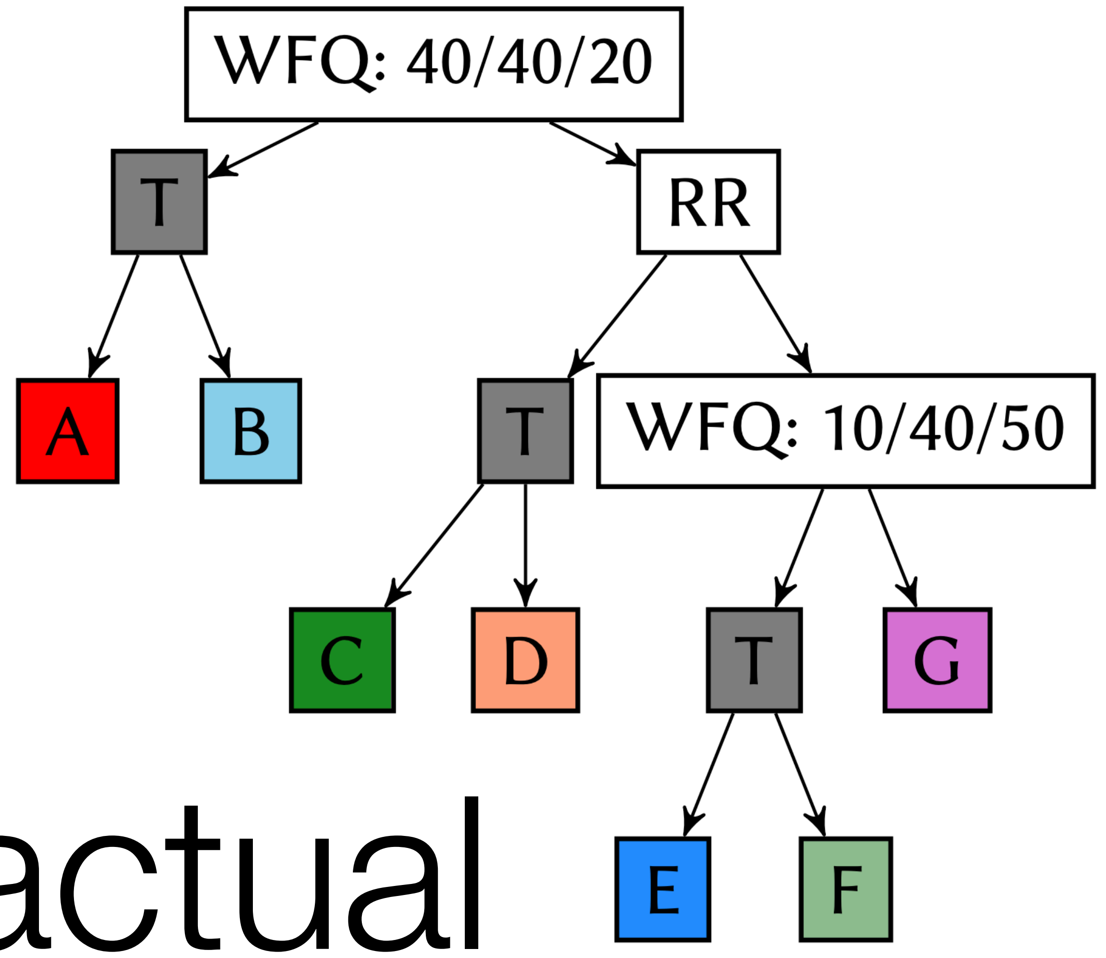
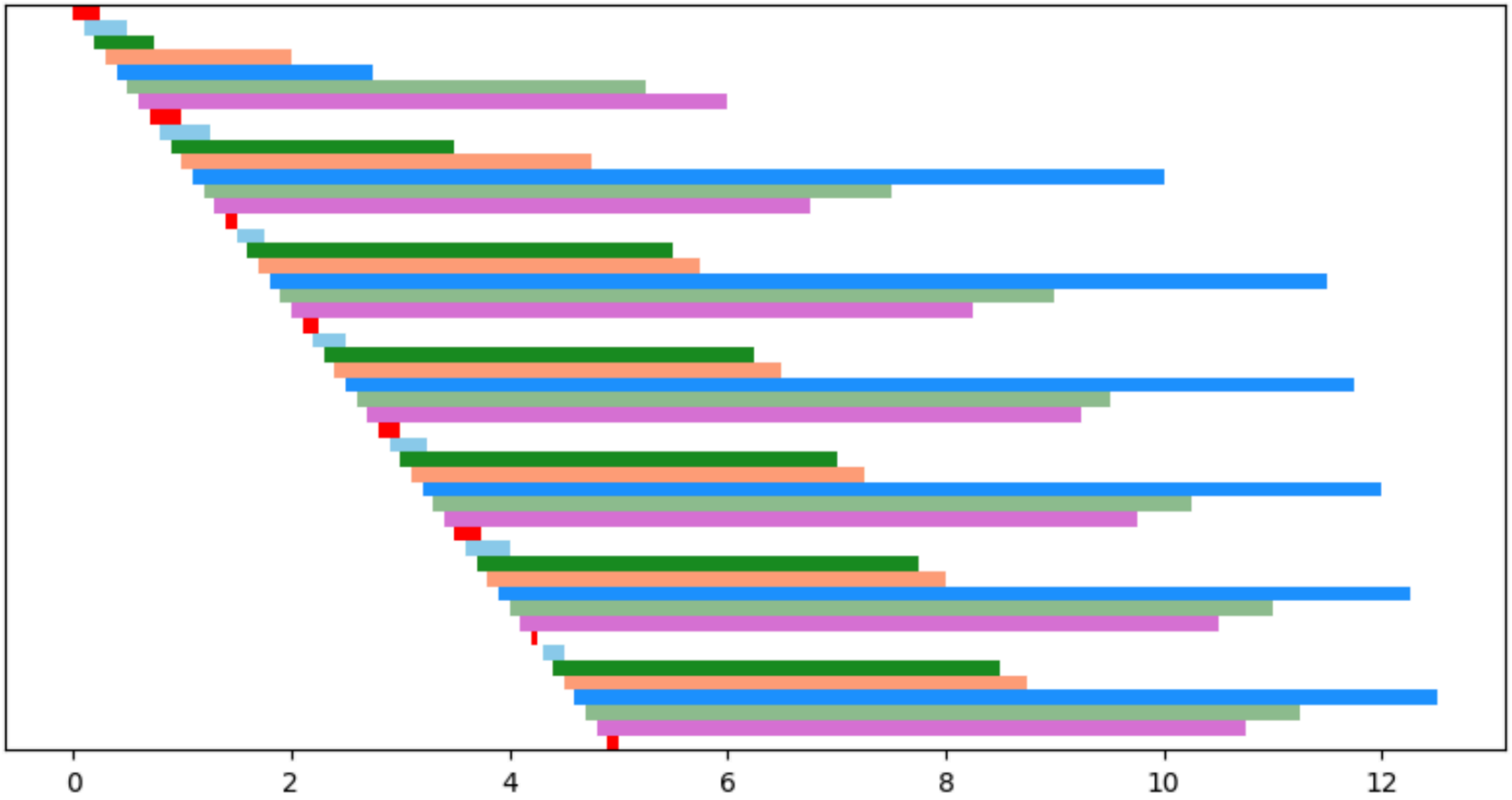


actual



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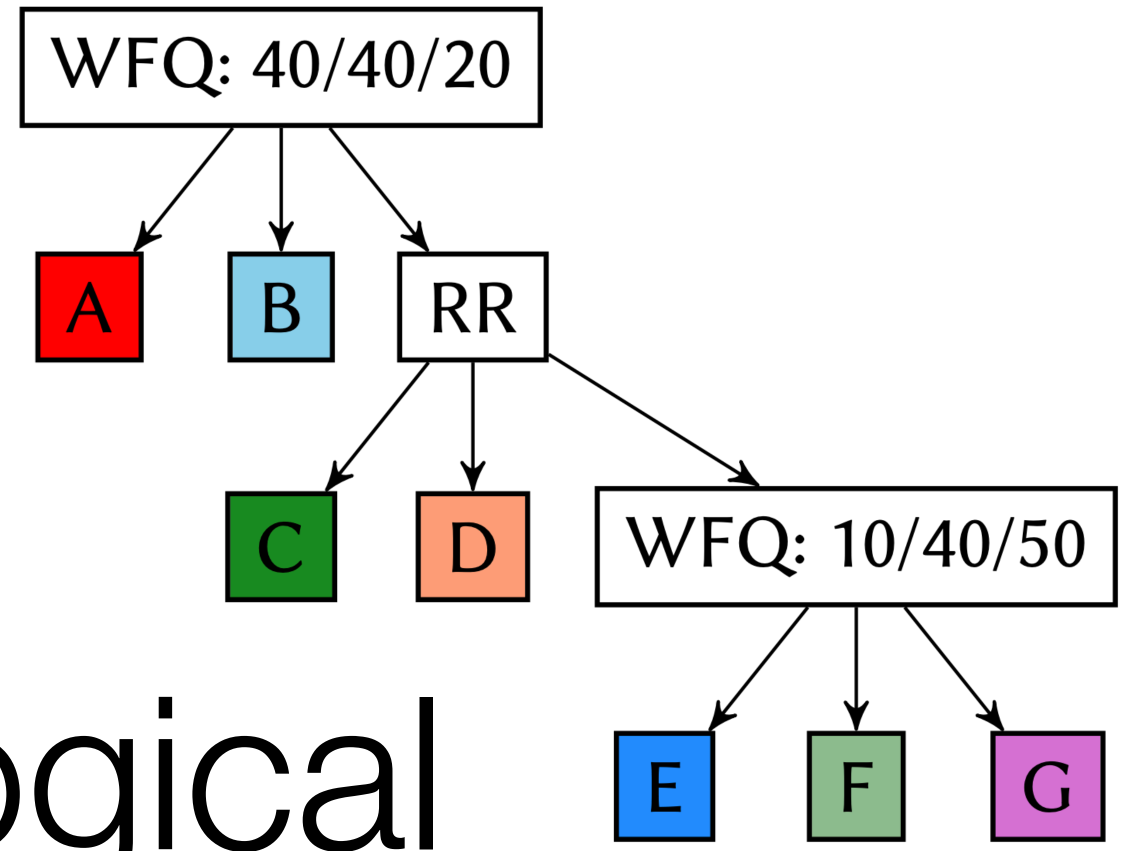

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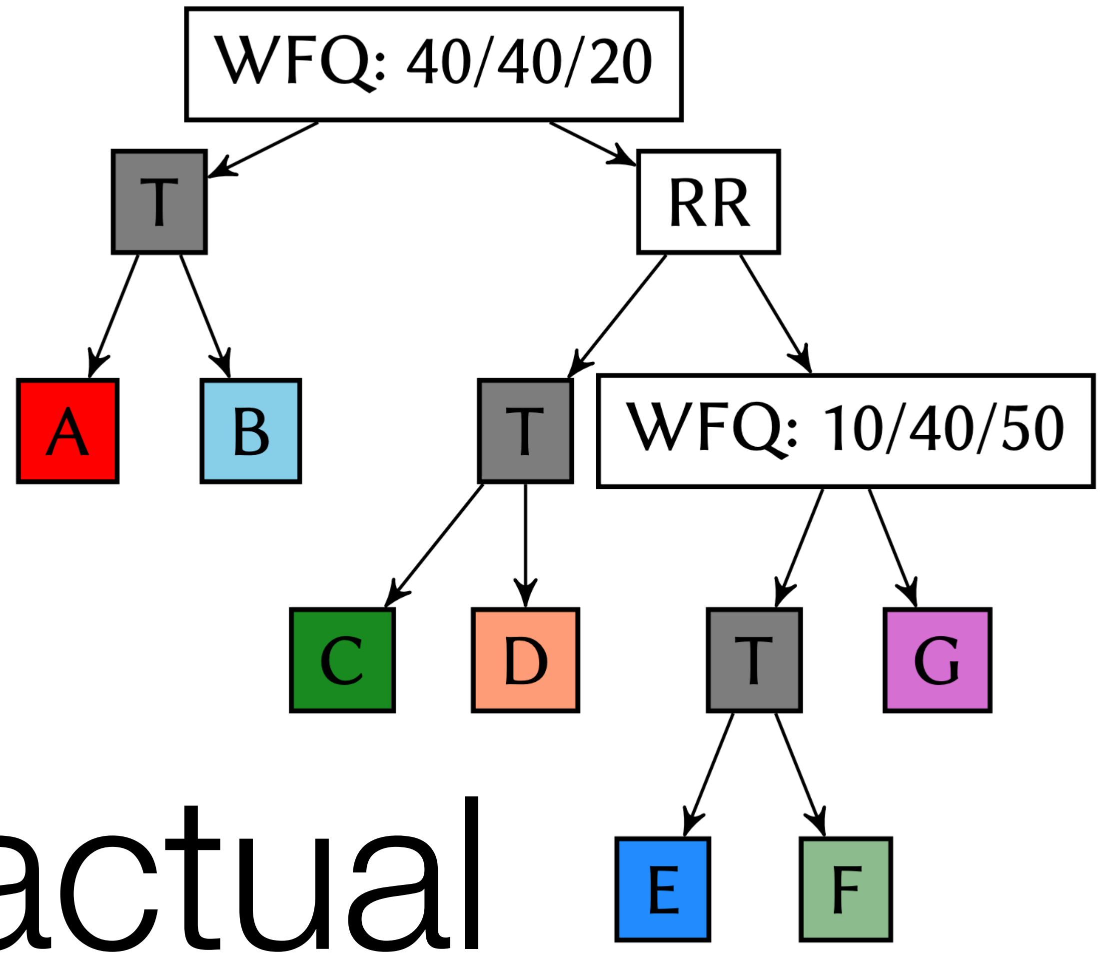

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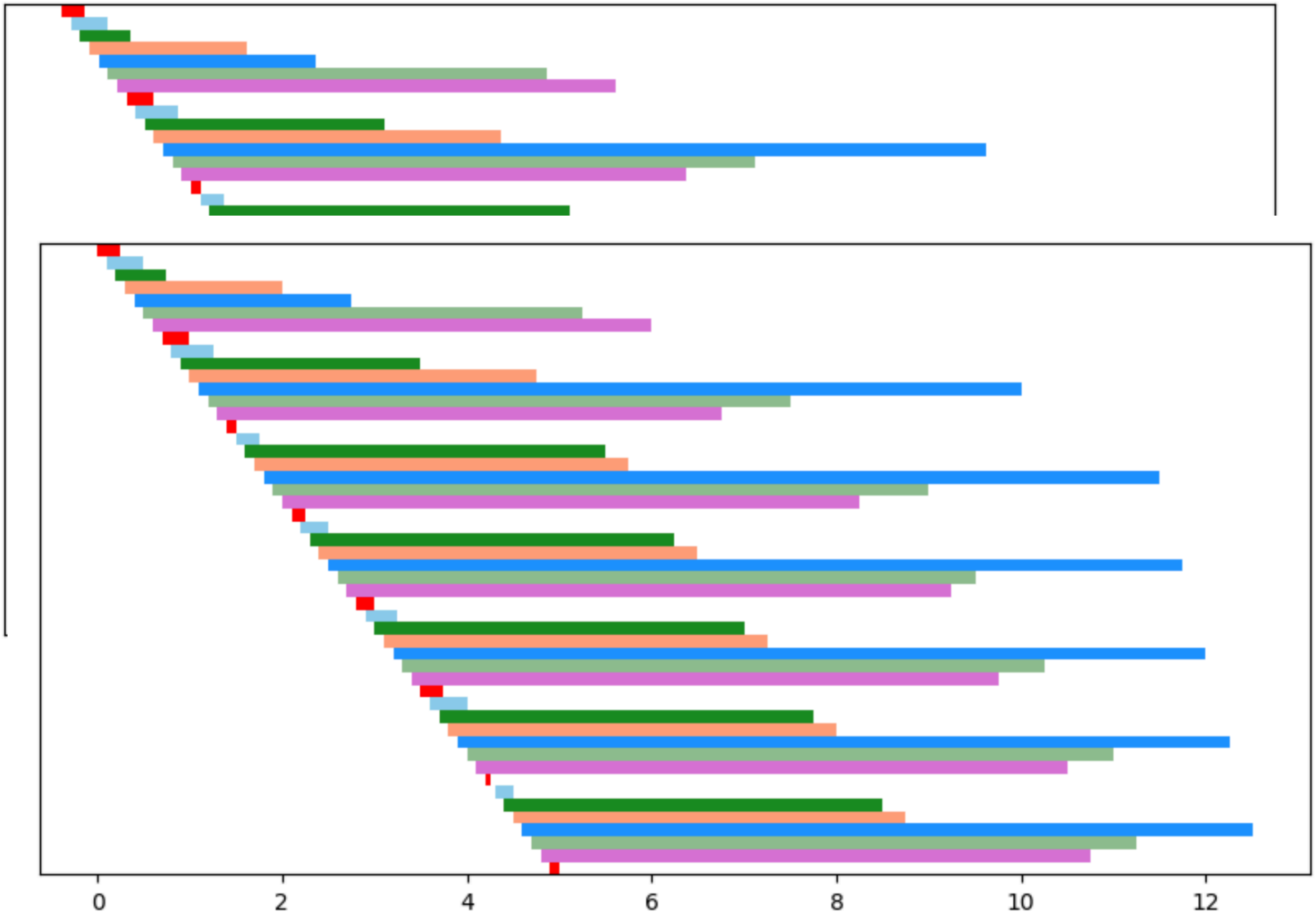

Underlying formalism

$$\frac{}{* \in \mathsf{Topo}} \qquad \frac{n \in \mathbb{N} \qquad ts \in \mathsf{Topo}^n}{\mathsf{Node}(ts) \in \mathsf{Topo}}$$

$$\frac{p \in \mathsf{PIFO}(\mathsf{Pkt})}{\mathsf{Leaf}(p) \in \mathsf{PIFOTree}\underbrace{(*)}_{\text{Topo}}} \qquad \frac{n \in \mathbb{N} \qquad ts \in \mathsf{Topo}^n \qquad p \in \mathsf{PIFO}(\{1, \ldots, n\}) \\ \forall 1 \leq i \leq n.\ qs[i] \in \mathsf{PIFOTree}(ts[i])}{\mathsf{Internal}(qs, p) \in \mathsf{PIFOTree}(\underbrace{\mathsf{Node}(ts)}_{\text{Topo}})}$$

$$\frac{r \in \mathsf{Rk}}{r \in \mathsf{Path}\underbrace{(*)}_{\text{Topo}}} \qquad \frac{ts \in \mathsf{Topo}^n \qquad 1 \leq i \leq n \qquad r \in \mathsf{Rk} \qquad pt \in \mathsf{Path}(ts[i])}{(i, r) :: pt \in \mathsf{Path}(\underbrace{\mathsf{Node}(ts)}_{\text{Topo}})}$$

$$\frac{\textsc{push}(p, pkt, r) = p'}{\mathsf{push}(\mathsf{Leaf}(p), pkt, r) = \underbrace{\mathsf{Leaf}(p')}_{\text{PIFOTree}}} \qquad \frac{\mathsf{push}(qs[i], pkt, pt) = q' \qquad \textsc{push}(p, i, r) = p'}{\mathsf{push}(\mathsf{Internal}(qs, p), pkt, \underbrace{(i, r) :: pt)}_{\text{Path}} = \underbrace{\mathsf{Internal}(qs[i/q'], p')}_{\text{PIFOTree}}}$$

A general way to deploy PIFO trees.

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Let the hardware support some tree.

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Let the human program against some tree.

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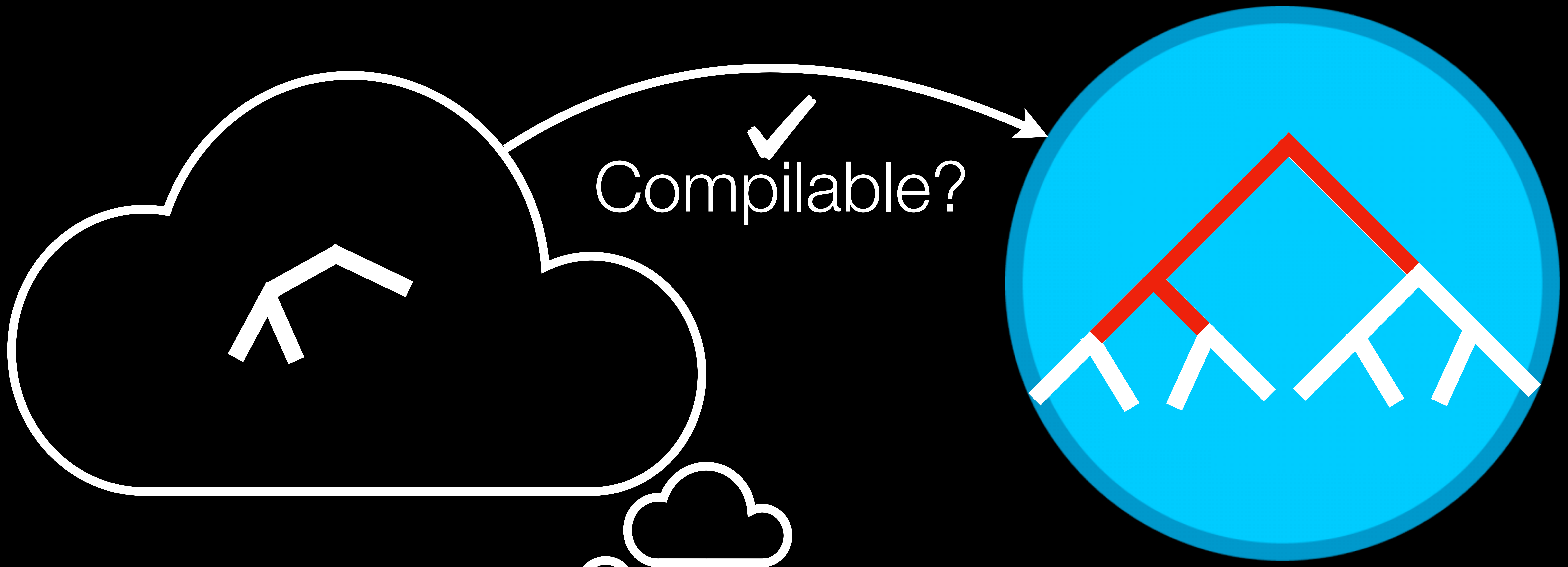

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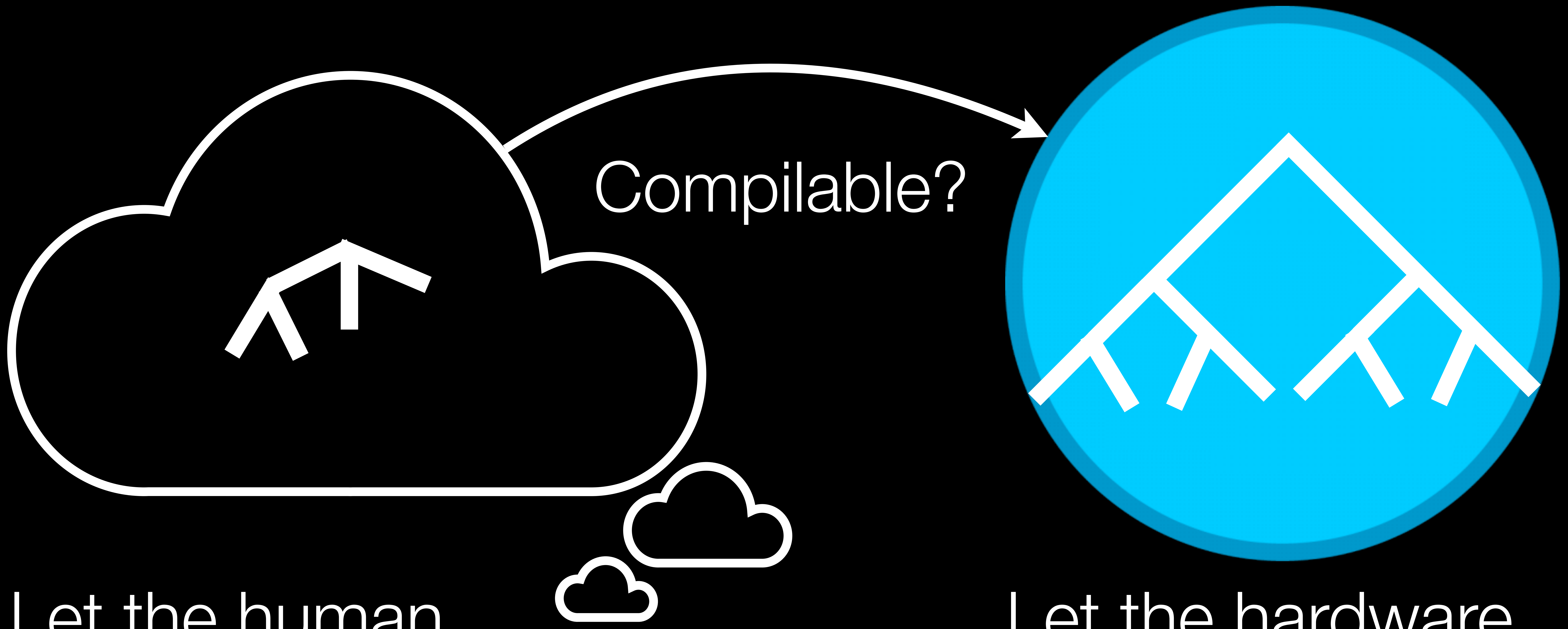

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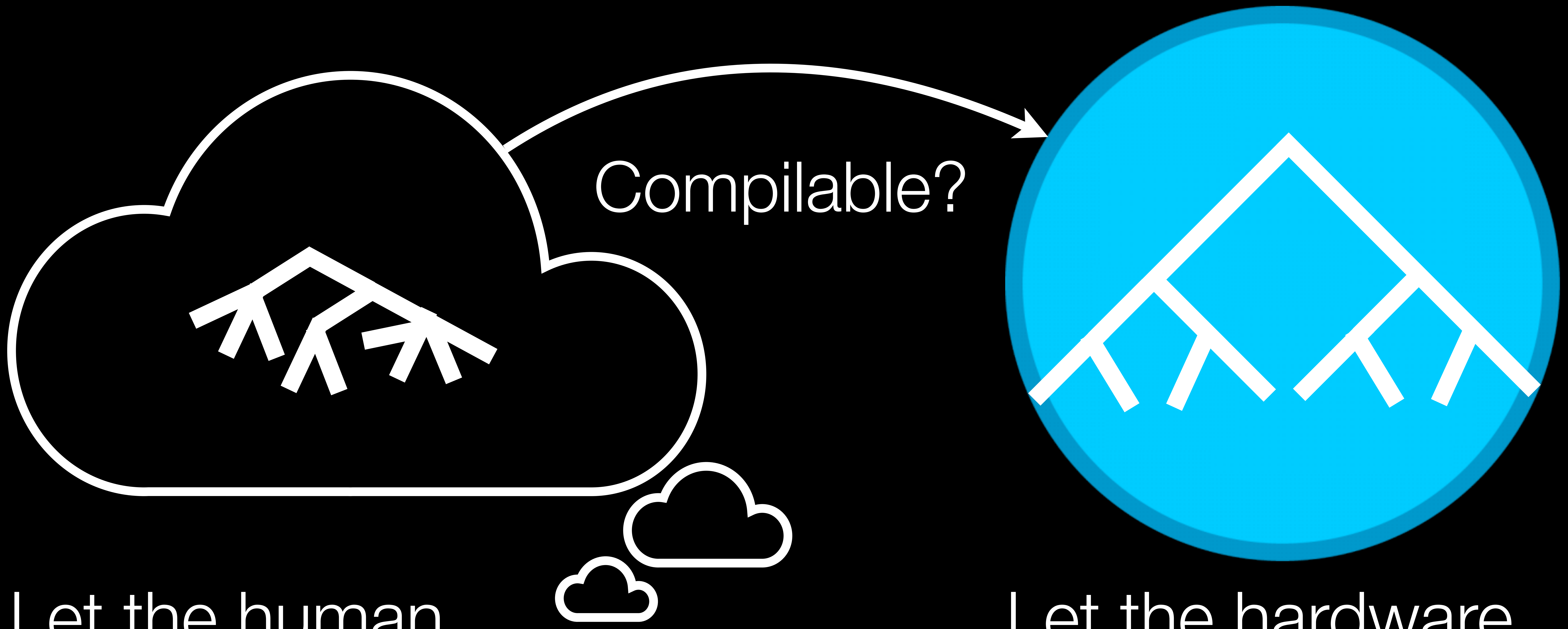

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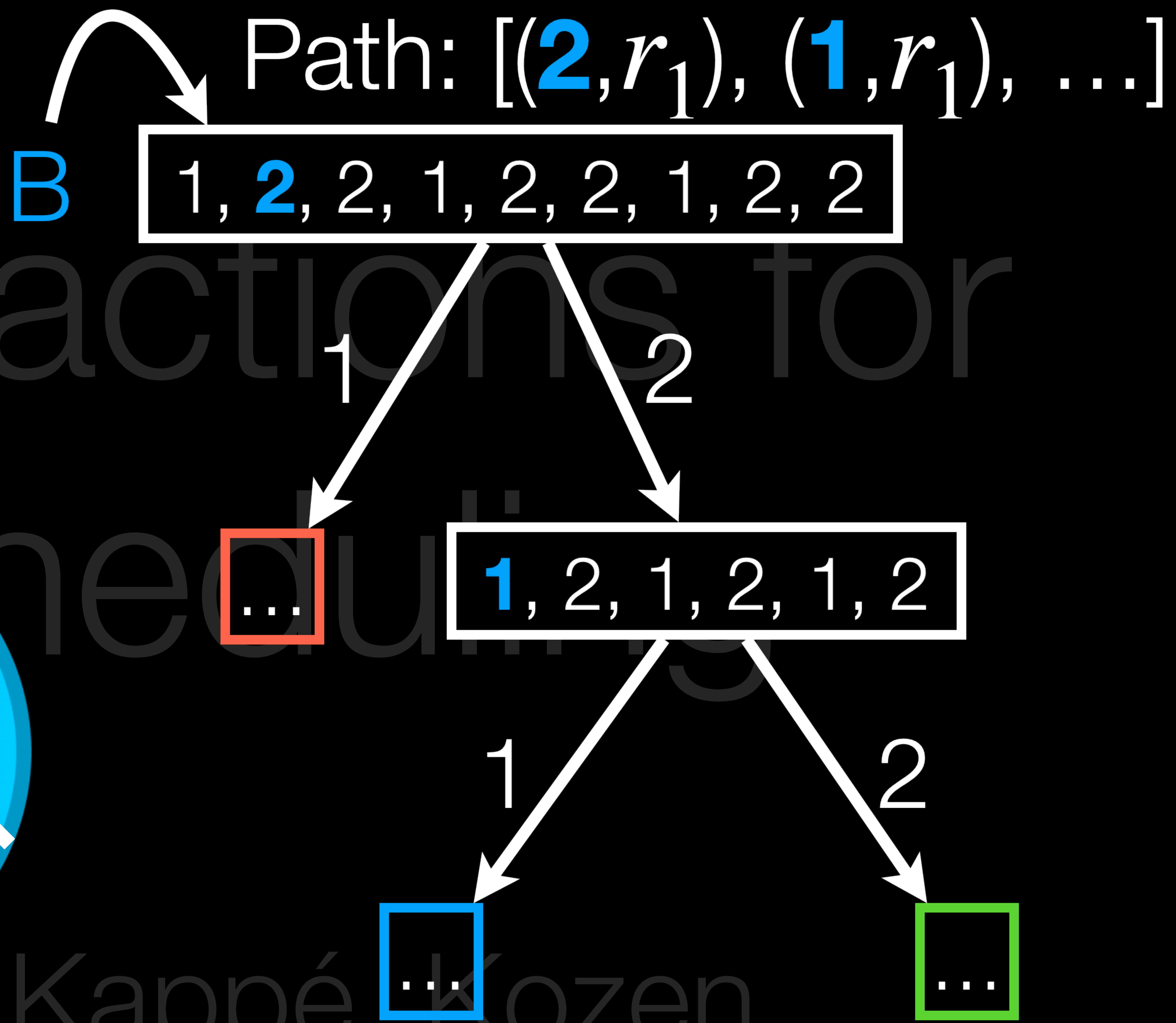
tree shape ⟷ language expressivity



Formal Abstractions for Packet Scheduling

Mohan, Liu, Foster, Kappé, Kozen

cs.cornell.edu/~amohan