Towards concurrent NetKAT

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CReNKAT kick-off workshop, 12/03/2019

Joint work with Paul Brunet, Bas Luttik, Jurriaan Rot, Alexandra Silva, Jana Wagemaker, Fabio Zanasi.
Introduction
Introduction

Towards concurrent NetKAT

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Introduction
What do you mean, concurrency?
What do you mean, concurrency?
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What do you mean, concurrency?
What do you mean, concurrency?
NetKAT plus parallel composition

\[
e, f ::= 0 \mid 1 \mid f \leftarrow v \mid f = v \mid e + f \mid e \cdot f \mid e^*
\]
NetKAT plus parallel composition

\[ e, f ::= 0 \mid 1 \mid f \leftarrow v \mid f = v \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]
NetKAT plus parallel composition

\[ e, f ::= 0 \mid 1 \mid f \leftarrow v \mid f = v \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

\[ e \parallel f \equiv f \parallel e \]
NetKAT plus parallel composition

\[ e, f ::= 0 \mid 1 \mid f \leftarrow v \mid f = v \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

\[ e \parallel f \equiv f \parallel e \quad e \parallel (f \parallel g) \equiv (e \parallel f) \parallel g \]
NetKAT plus parallel composition

\[ e, f ::= 0 | 1 | f \leftarrow v | f = v | e + f | e \cdot f | e^* | e \parallel f \]

\[ e \parallel f \equiv f \parallel e \quad e \parallel (f \parallel g) \equiv (e \parallel f) \parallel g \quad e \parallel 0 \equiv 0 \]
NetKAT plus parallel composition

\[ e, f ::= 0 | 1 | f \leftarrow v | f = v | e + f | e \cdot f | e^* | e \parallel f \]

\[ e \parallel f \equiv f \parallel e \quad e \parallel (f \parallel g) \equiv (e \parallel f) \parallel g \quad e \parallel 0 \equiv 0 \quad e \parallel 1 \equiv e \]
NetKAT plus parallel composition

\[(e \cdot f) \parallel (g \cdot h)\]

Thread 1

Thread 2

\[\begin{array}{c}
e \\
g \\
f \\
h \end{array}\]
NetKAT plus parallel composition

\[(e \parallel g)(f \parallel h)\]

Thread 1

\hspace{1cm} e \hspace{1cm} f

Thread 2

\hspace{1cm} g \hspace{1cm} h
NetKAT plus parallel composition

\[(e \parallel 1)(1 \parallel h)\]

Thread 1

\(e\) \hspace{1cm} \(f\)

Thread 2

\(g\) \hspace{1cm} \(h\)
NetKAT plus parallel composition

Why not do total interleaving?
- Requires synchronizing packet state across nodes.
- Individual copies may be modified along the way.
NetKAT plus parallel composition

NetKAT

concurrency

“CNetKAT”
NetKAT plus parallel composition

KA \xrightarrow{tests} \text{KAT} \xrightarrow{networks} \text{NetKAT}

\text{“CNetKAT”}

\text{concurrency}
NetKAT plus parallel composition

See e.g. [HMS⁺]
NetKAT plus parallel composition

\[ \text{KA} \xrightarrow{\text{tests}} \text{KAT} \xrightarrow{\text{networks}} \text{NetKAT} \]

\[ \text{CKA} \xrightarrow{\text{tests}} \text{CKAT} \]

\text{See e.g. [JM]}
NetKAT plus parallel composition

\[ \text{KA} \xrightarrow{\text{tests}} \text{KAT} \xrightarrow{\text{networks}} \text{NetKAT} \]

\[ \text{CKA} \xrightarrow{\text{tests}} \text{CKAT} \xrightarrow{\text{networks}} \text{“CNetKAT”} \]
Concurrent Kleene Algebra

\[ a \cdot b \approx a \rightarrow b \]
Concurrent Kleene Algebra

\[ a \cdot b \cong a \rightarrow b \]

\[ a \parallel b \cong \]

\[ b \]

\[ a \]
Concurrent Kleene Algebra

\[ a \cdot b \approx a \rightarrow b \]

\[ c \cdot (a \parallel b) \approx c \bullet a \bullet b \]

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Concurrent Kleene Algebra

\[ a \cdot b \approx a \rightarrow b \]

\[ c \cdot (a \parallel b) \cdot d \approx \]

\[ \begin{array}{c}
\xymatrix{c \ar[r] & b \\
\ar[r] & a \\
\ar[r] & \ar[u] & d 
} \end{array} \]
Concurrent Kleene Algebra

\[ a \cdot b \approx a \rightarrow b \]

\[ c \cdot (a \parallel b) \cdot d \approx a \rightarrow b \]

\[ a 
\rightarrow b \subseteq a \quad b \]
Concurrent Kleene Algebra

\[ a \cdot b \approx a \rightarrow b \]

\[ c \cdot (a \parallel b) \cdot d \approx c \rightarrow d \]

\[ a \rightarrow b \sqsubseteq a \parallel b \]

\[ a \rightarrow c \sqsubseteq a \parallel c \]

\[ b \rightarrow d \sqsubseteq b \parallel d \]
Concurrent Kleene Algebra

\[ e, f ::= 0 \mid 1 \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \]
Concurrent Kleene Algebra

\[ e, f ::= 0 \mid 1 \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]
Concurrent Kleene Algebra

\[ e, f ::= 0 \mid 1 \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

\[
\begin{align*}
[0] &= \emptyset \\
[1] &= \{1\} \\
[a] &= \{a\} \\
[e^*] &= [e]^* \downarrow \\
[e + f] &= [e] \cup [f] \\
[e \cdot f] &= [e] \cdot [f] \\
[e \parallel f] &= ([e] \parallel [f]) \downarrow
\end{align*}
\]
Concurrent Kleene Algebra

\[ e, f ::= 0 \mid 1 \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

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\begin{align*}
[0] &= \emptyset \\
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[e^*] &= [e]^* \downarrow \\
[e + f] &= [e] \cup [f] \\
[e \cdot f] &= [e] \cdot [f] \\
[e \parallel f] &= ([e] \parallel [f]) \downarrow
\end{align*}
\]

pairwise
Concurrent Kleene Algebra

Theorem [BPS]

Given terms \(e\) and \(f\), it is decidable whether \([e] = [f]\).

Theorem [KBS⁺]

Given terms \(e\) and \(f\), we have that \(e \equiv f\) if and only if \([e] = [f]\).
Concurrent Kleene Algebra
Concurrent Kleene Algebra

\[ q_0 \xrightarrow{a} q_1 \xrightarrow{c} q_4 \xrightarrow{b} q_3 \xrightarrow{b} q_5 \xrightarrow{e} q_2 \xrightarrow{e} q_7 \xrightarrow{d} q_6 \xrightarrow{d} q_8 \]
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Concurrent Kleene Algebra

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Concurrent Kleene Algebra

\[ q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_5 \]

\[ q_1 \xrightarrow{b} q_3 \xrightarrow{c} q_4 \]

\[ q_2 \xrightarrow{e} q_7 \]

\[ q_4 \xrightarrow{c} q_6 \xrightarrow{d} q_8 \]
Concurrent Kleene Algebra

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12 | 19
Concurrent Kleene Algebra

\[
\begin{array}{cccc}
 q_0 & \xrightarrow{a} & q_1 & \xrightarrow{e} q_2 \\
 q_3 & \xrightarrow{b} & q_5 \\
 q_4 & \xrightarrow{c} & q_6 & \xrightarrow{d} q_8
\end{array}
\]
Concurrent Kleene Algebra

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Theorem \([\text{KBS}^+; \text{KBL}^+]\)

Let \(L\) be a pomset language. The following are equivalent:

- \(L\) is recognized by a series-rational expression \(e\).
- \(L\) is recognized by a fork-acyclic and well-structured pomset automaton.
Theorem [KBS⁺; KBL⁺]

Let $L$ be a pomset language. The following are equivalent:

- $L$ is recognized by a series-rational expression $e$.
- $L$ is recognized by a fork-acyclic and well-structured pomset automaton.

Theorem [KBL⁺]

Language equivalence of fork-acyclic and well-structured pomset automata is decidable.
Concurrent Kleene Algebra... with tests?

\[ e, f ::= a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in O \mid p \lor q \mid p \land q \mid \bar{p} \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in \emptyset \mid p \lor q \mid p \land q \mid \overline{p} \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

\[ p + q \equiv p \lor q \quad p \land q \equiv p \cdot q \quad p \land \overline{p} \equiv 0 \]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in O \mid p \lor q \mid p \land q \mid \bar{p} \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

\[ p + q \equiv p \lor q \quad p \land q \equiv p \cdot q \quad p \land \bar{p} \equiv 0 \]

\[ p \cdot e \cdot \bar{p} \]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in O \mid p \lor q \mid p \land q \mid \neg p \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

\[ p + q \equiv p \lor q \]

\[ p \land q \equiv p \cdot q \]

\[ p \land \neg p \equiv 0 \]

\[ p \cdot e \cdot \neg p \leq (p \cdot \neg p) \parallel e \]
Concurrent Kleene Algebra... with tests?

\[
\begin{align*}
p, q & ::= 0 \mid 1 \mid o \in \mathcal{O} \mid p \lor q \mid p \land q \mid \overline{p} \\
e, f & ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f
\end{align*}
\]

\[
\begin{align*}
p + q & \equiv p \lor q & p \land q & \equiv p \cdot q & p \land \overline{p} & \equiv 0 \\
p \cdot e \cdot \overline{p} & \leq (p \cdot \overline{p}) \parallel e & e & \equiv (p \land \overline{p}) \parallel e
\end{align*}
\]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in \mathcal{O} \mid p \lor q \mid p \land q \mid \neg p \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f \]

\[ p + q \equiv p \lor q \quad p \land q \equiv p \cdot q \quad p \land \neg p \equiv 0 \]

\[ p \cdot e \cdot \neg p \leq (p \cdot \neg p) \parallel e \equiv (p \land \neg p) \parallel e \equiv 0 \parallel e \]
Concurrent Kleene Algebra... with tests?

\[
p, q ::= 0 \mid 1 \mid o \in \mathcal{O} \mid p \lor q \mid p \land q \mid \overline{p}
\]

\[
e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \mid e \parallel f
\]

\[
p + q \equiv p \lor q \quad p \land q \equiv p \cdot q \quad p \land \overline{p} \equiv 0
\]

\[
p \cdot e \cdot \overline{p} \leq (p \cdot \overline{p}) \parallel e \equiv (p \land \overline{p}) \parallel e \equiv 0 \parallel e \equiv 0
\]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in \mathcal{O} \mid p \lor q \mid p \land q \mid \overline{p} \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in O \mid p \lor q \mid p \land q \mid \overline{p} \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \]

\[ p + q \equiv p \lor q \quad p \land q \leq p \cdot q \quad p \land \overline{p} \equiv 0 \]
Concurrent Kleene Algebra... with tests?

\[ p, q ::= 0 \mid 1 \mid o \in \mathcal{O} \mid p \lor q \mid p \land q \mid \bar{p} \]

\[ e, f ::= p \mid a \in \Sigma \mid e + f \mid e \cdot f \mid e^* \]

\[ p + q \equiv p \lor q \quad p \land q \leq p \cdot q \quad p \land \bar{p} \equiv 0 \]

Theorem preprint: [KBR⁺]

There is a language semantics \([-\cdot-]\) such that

1. \([e] = [f]\) if and only if \(e \equiv f\), and
2. it is decidable whether \([e] = [f]\).
Concurrent Kleene Algebra... with tests?

KA
Concurrent Kleene Algebra... with tests?

\[ KA \xrightarrow{\text{observations}} KAO \]
Concurrent Kleene Algebra... with tests?

\[ p \cdot q \equiv p \land q \]

KA → observations → KAO

concurrency

 ↙

CKA
Concurrent Kleene Algebra... with tests?

\[
[p \cdot q] \equiv [p \land q]
\]

Ka-Observations

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Concurrent Kleene Algebra... with tests?

\[ [p \cdot q] \equiv [p \land q] \]
Hurdles on the horizon

Multicasting or nondeterminism?
Hurdles on the horizon

Multicasting or nondeterminism?

\[ e + f \equiv f \iff e \leq f \]
Hurdles on the horizon

True concurrency versus sequential consistency?
Hurdles on the horizon

True concurrency versus sequential consistency?

\[(f \leftarrow v) \cdot (f' \leftarrow v') \equiv (f' \leftarrow v') \cdot (f \leftarrow v)\]


Peter Jipsen and M. Andrew Moshier. Concurrent Kleene algebra with tests and branching automata. DOI: 10.1016/j.jlamp.2015.12.005.


Tobias Kappé et al. Kleene algebra with observations. eprint: 1811.10401.

Tobias Kappé et al. Concurrent Kleene algebra: free model and completeness. In DOI: 10.1007/978-3-319-89884-1_30.